Journal of civil engineering and transport **transEngin**

2022, Volume **4** Number **3** **ISSN 2658-1698, e-ISSN 2658-2120**

DOI: 10.24136/tren.2022.010

ECONOMETRIC MODELS – A METHOD FOR EXAMINING FACTORS OF IMPLEMENTATION OF PUBLIC-PRIVATE PARTNERSHIP PROJECTS IN SELECTED EUROPEAN COUNTRIES

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Reviewed positively: 26.09.2022

Information about quoting an article:
Rogowski A., Zagożdżon B. (2022). Econometric models – a method for examining factors of implementation of public-private partnership projects in selected European countries. Journal of civil engineering and transport. 4(3), 25-55, ISSN 2658-1698, e-ISSN 2658-2120, DOI: 10.24136/tren.2022.010

Abstract – Modern science is based on the study of economic phenomena and tries to quantify them in a measurable way. Econometric models are used for this purpose. The object of this research was to develop econometric models that show the strength of the influence of various factors on the implementation of public-private partnership (PPP) projects in the area of transport infrastructure in France, GB, Germany, the Netherlands and Belgium. The models express the dependence of the value and number of PPP contracts on the value of measurable PPP success factors. Projects with a value of at least €40 million were included. A linear model and seven models transformable to linear were used. Four groups of factors were considered as explanatory variables. Fourteen indicators were obtained. Principal components determined based on covariance and correlation matrices were also used. The best models for the number of PPP contracts are linear and hyperbolic I models. For the value of contracts – linear and hyperbolic I and logarithmic models. The best models were indicated taking into account the type of explanatory variables and regardless of the type of explanatory variables. Nine criteria were used to assess the quality of the models. Factors having a significant impact on the value and number of PPP models were identified from the best models. Factors having no significant influence were also indicated.

Key words – econometric model, linear regression, non-linear regression, public-private partnership PPP

JEL Classification – R41, R42, R15

INTRODUCTION

One of the main problems of transport is the development of the infrastructure which determines a country's economic and social activity. Europe needs a well-developed, efficient transport network to support economic development, trade and job creation, while pursuing a policy of sustainable development. The construction of transport infrastructure is an investment challenge for EU countries, especially those adopted in 2004 and beyond. In these countries, transport infrastructure is technically inadequate to meet current transport needs and requires high levels of investment. Transport infrastructure has specific characteristics, such as service and social character, capital intensity, long payback period, technical and economic indivisibility [1]. These characteristics meant that infrastructure was widely considered to be the domain of government action and funding. However, in recent decades, economists working on this issue present the view that infrastructure is no longer seen as a public good and should not be provided solely by the state [2-3]. It is now believed that there are no goods and services that can only be provided by the private or public sector [4]. When looking for alternative sources of financing transport investments, in relation to financing from the state budget, attention was drawn to the public-private partnership (PPP) formula.

However, the practice of PPP implementation and the conducted research show that the success of the partnership requires appropriate economic, legal, institutional and social conditions [5-6]. Moreover, recent reports of the EU institution, which has been supporting the development of PPPs for several years, have shown that the implementation of successful projects is possible with the appropriate legal, institutional framework and administrative efficiency. These instruments are currently only available in a limited number of countries that have years of experience in implementation [7].

An important research issue in the question of PPP is the identification of factors of projects' success and determining the influence of particular factors on the process of their successful implementation. The aim of the research undertaken was to develop and verify econometric models that make it possible to determine the strength of the impact of individual factors on the implementation of PPP projects in selected European countries. The developed models express the dependence of the value and number of PPP agreements on the value of measurable factors of success of PPP in Great Britain, France, Germany, Belgium and the Netherlands. Transport projects with a value of no less than €40 million were considered. Four groups of these factors were considered as independent variables: economic and financial, political and legal, technical and social. These groups of factors were given 14 indicators. The next stage of the research was the analysis and evaluation of the usefulness of the models for determining the strength of the influence of particular factors on the implementation of PPP projects.

The research objective formulated in this way also resulted from the fact that in the conducted literature studies, there were no works in which mathematical models expressing the examined relations were constructed. This research issue is presented in this article.

1. REVIEW OF APPLIED RESEARCH METHODS

A public-private partnership, on the basis of literature research, can be defined as a type of agreement between a public and a private partner, who act together in the implementation of a project while maintaining their own objectives and interests [8-9]. The result of this cooperation should be a lower cost of the undertaking and a higher quality of services than if they were financed in a traditional way – with public funds. The form of involvement of private entities in PPP projects depends on its specificity and individual needs of project participants. A detailed classification of the forms of PPP was presented, among others, in [10].

Conditions conducive to the implementation of PPPs are referred to as critical success factors – CSFs. Their identification was the subject of an extensive study [11], which identified five main groups of factors: effective procurement process, project feasibility, government guarantee, favourable economic conditions and available financial market. A classification of CSF, which is based on different aspects of risk associated with PPP projects, was developed in [12]. A comprehensive overview of CSF is also presented in [13], The authors examined the importance of these factors, taking into account public, private and social sector objectives. They proposed dividing the factors into four groups: political and legal, economic and financial, technical and social. In the case of transport infrastructure projects, an important factor pointed out by some authors [5, 14] is the stable macroeconomic situation of the country, especially indicators such as GDP growth, purchasing power of customers, market size.

The main research problem was the choice of research methodology on the basis of which the strength of the influence of individual CSF factors on the implementation of PPP projects will be determined. Thus, the classification of papers in terms of the methodologies used was the subject of a study conducted in [15]. The authors reviewed 85 papers. The case study was used most often, which is probably due to the fact that it is easier to draw specific conclusions based on real cases than other research methods. Survey and literature review came next, followed by interview. More than half of them required the collection of primary data and their statistical processing.

Statistical elaboration takes many forms, from the simplest tabulation and charting to the use of more advanced statistical methods.

One of the frequently used methods is surveys. Their purpose is usually to identify, classify and evaluate the success factors of PPPs. In a paper [16], the authors conducted a survey with 108 responses from experts, researchers and project managers. The fuzzy synthetic evaluation (FSE) method was then applied and nine most important factors were obtained. These were: private sector financial capacity, government credit, government guarantee, appropriate legal framework, available financial markets, feasibility study report and implementation, risk management effectiveness, project investment, and cost control and revenue sharing. Other studies have compared the use of PPPs in high-speed rail (HSR) projects with infrastructure projects in general. The authors [14] developed a structured questionnaire for professionals. They used a classic 5-point Likert scale. They used mean value analysis (MVA), confirmatory factor analysis (CFA) and Cronbach's alpha reliability test. The paper [11] analysed the relative importance of eighteen critical success factors for PPP/PFI (Private Finance Initiative) in construction projects in the GB. Likert scale and statistical analyses including descriptive analysis, Cronbach's alpha reliability tests, one-way analysis of variance and factor analysis were used. Factor analysis was used in [17]. At the same time, they used principal component analysis and the Varimax method with Kaiser normalisation. Their study aimed to identify the critical success factors of PPPs under transition conditions.

In [18], the research team applied the management concept and the systems approach. The aim of the research was to synthesise evidence on the complexity of the PPP formation phase in the road sector. A textual analysis of case studies described in papers in Scopus and Web of Science between 1997 and 2018 was applied. A system dynamics approach was also used to provide a holistic view, synthesising the main insights and the arrangement of interdependencies between financial, operational and socio-political variables.

Besides statistical methods, other mathematical tools are also used. In the paper [19] quite advanced methods of probability calculus were used. At the same time, the optimal (from a social point of view) toll, road quality and concession duration were compared. In contrast, [20] used game theory to analyse the process of risk sharing between the public and private sectors in transport infrastructure contracts. He conducted this research based on an arbitrage game with a final offer.

2. Model building methodology

The process of model building and analysis of the impact of measurable success factors on the number (L) and value (W) of PPP contracts in each country was carried out in two stages. In the first stage, a general linear model and non-linear models (logarithmic, power, exponential, log-hyperbolic and three hyperbolic models, Table 1) were used to build regression models expressing the value (W) and number (L) of PPP contracts concluded in a given year.

Table 1. General linear regression model and non-linear models

Model	Equation
linear	$y = \sum_{i=1}^{n} a_i x_i + b$
power	$y = \exp\left(\sum_{i=1}^{n} a_i \ln x_i + b\right)$
exponential	$y = \exp\left(\sum_{i=1}^{n} a_i x_i + b\right)$
logarithmic	$y = \sum_{i=1}^{n} a_i \ln x_i + b$
log-hyperbolic	$y = \exp\left(\sum_{i=1}^{n} \frac{a_i}{x_i} + b\right)$
hyperbolic I	$y = \sum_{i=1}^{n} \frac{a_i}{x_i} + b$
hyperbolic II	$y = \left(\sum_{i=1}^{n} a_i x_i + b\right)^{-1}$
hyperbolic III	$y = \left(\sum_{i=1}^{n} \frac{a_i}{x_i} + b\right)^{-1}$

All calculations were performed in an Excel spreadsheet. The parameters of the models were calculated using the REGLINP function (for nonlinear models after transformation to linear form). Four groups of factors were considered as independent (explanatory) variables: economic-financial, political-legal, technical and social (Tables 2 ÷ 6), resulting in 14 indicators.

Table 2. Value and number of PPP agreements concluded in a given year between 2009-2019 in France and selected macroeconomic indicators

D	Years										
	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
W	0	0.93	8	1.9	0.58	0.2	0.99	0.62	0.03	0	0.64
L	0	5	7	3	4	3	2	2	1	0	2
1	1936.42	1995.29	2058.37	2088.8	2117.19	2149.77	2198.43	2234.13	2297.24	2360.69	2425.71
2	348.04	395.09	428.5	442.64	437.15	437.79	456.51	453.08	473.81	492.96	509.95
3	138.93	137.41	106.1	104.04	86.47	83.94	79.7	81.26	67.96	54.1	74.71
4	1607.98	1701.12	1807.96	1892.54	1977.73	2039.88	2101.26	2188.48	2259.62	2314.9	2379.5
5	-287.3	-185.2	-178.4	-267.8	-350.5	-335.7	-284.2	-290.6	-462.1	-444.6	-556.5
6	-14.8	-9.3	-8.7	-12.8	-16.6	-15.6	-12.9	-13	-20.1	-18.8	-22.9
7	3.3	4.6	6.4	4.2	2.1	3.2	4.6	6.2	7.1	8.2	8
8	165	166	171	176	174	187	193	199	201	206	212
9	2168.79	2234.73	2305.37	2381.23	2413.60	2429.24	2462.24	2502.23	2595.88	2643.97	2716.80
10	112	112	112	114	114	113	112	112	113	112	112
11	8.07	7.77	7.8	7.88	7.92	8.04	7.92	7.92	7.8	7.8	8.12
12	0.73	0.74	0.7	0.78	0.78	0.79	0.74	0.72	0.74	0.74	0.73
13	69	68	70.0	71	71	69	70	69	70	72	69
14	64.71	65.03	65.3	65.66	66	66.31	66.55	66.72	66.86	66.97	67.06

D – designation: **W** – Value of contracts concluded in a given year [bln EUR], **L** – Number of contracts concluded in a given year, (x_i): **1** – Gross Domestic Product GDP (bln EUR), **2** – Exports (bln EUR), **3** – Deficit (bln EUR), **4** – Public Debt (bln EUR), **5** – International Investment Position (bln EUR), **6** – International Investment Position (% GDP), **7** – Private Sector Credit (% GDP), **8** – Private Sector Debt (% GDP), **9** – Bank Assets (bln EUR), **10** – Bank Assets (% GDP), **11** – Democracy Index, **12** – Rule of Law Index, **13** – Corruption Index, **14** – Population (mln)

Own study based on Eurostat, European Commission.

Table 3. Value and number of PPP agreements concluded in a given year between 2009-2019 in the Great Britain and selected macroeconomic indicators

D	Years										
	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
W	1.96	0.38	0.62	3.77	2.11	4.68	**0**	**0**	0.89	**0**	1.42
L	3	1	1	4	1	4	0	0	1	0	1
1	1738.07	1872.18	1912.87	2111.03	2096.34	2311.08	2644.72	2434.12	2359.79	2420.9	2526.62
2	254.7	313.77	363.92	367.99	407.06	380.19	414.7	369.9	390.72	412.06	419.8
3	172.22	173.23	141.14	170.24	114.18	127.81	112.17	99.48	65.76	66.84	48.84
4	1103.25	1387.56	1590.78	1745.86	1799	2060.36	2269.87	2022.24	2063.52	2116.08	2185.08
5	-322.1	-172.2	-258.8	-606	-402.4	-523.6	-581.3	-45	-346.1	-388.4	-764.9
6	-18.5	-9.2	-13.5	-28.7	-19.2	-22.7	-22	-1.8	-14.7	-16	-30.3
7	-7.7	-2.7	-1.2	0.5	3.9	2.3	0.8	5.8	4.9	3.6	2.9
8	190	182	178	178.5	171	163.5	161	160	162	161	156
9	3389.24	3500.98	3385.78	3462.09	3228.36	3258.62	3491.03	3164.36	3114.92	3171.38	3335.14
10	195	187	177	164	154	141	132	130	132	131	132
11	8.15	8.16	8.2	8.21	8.31	8.31	8.31	8.36	8.53	8.53	8.52
12	0.8	0.8	0.8	0.79	0.79	0.81	0.78	0.81	0.81	0.81	0.8
13	77	76	78	74	76	78	81	81	82	80	77
14	62.04	62.51	63.02	63.5	63.91	64.35	64.81	65.38	65.84	66.27	66.65

Designations as in Table 2

Table 4. Value and number of PPP agreements concluded in a given year between 2009-2019 in Germany and selected macroeconomic indicators

D	Years										
	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
W	0.57	0	0.78	0	0.01	0.8	**0.02**	**1.11**	0.42	**0.65**	1.3
L	3	0	2	0	1	2	1	3	1	1	1
1	2445.73	2564.4	2693.56	2745.31	2811.35	2927.43	3026.18	3134.74	3259.86	3356.41	3449.05
2	803.01	949.63	1058.9	1090.53	1088.07	1125.03	1195.82	1205.49	1281.95	1320.73	1330.41
3	77.05	112.29	23.74	-0.26	-1.12	-16.97	-29.07	-36.37	-44.4	-61.65	-52.47
4	1789.21	2114.89	2151.49	2229.14	2214.37	2216.2	2189.12	2172.33	2122.86	2074.13	2057.63
5	614.3	661.7	627.1	787.2	974.7	1934.7	1410	1609.7	1836.5	2116.8	2479.5
6	25.1	25.8	23.3	28.7	34.7	66.1	46.6	51.4	56.3	63.1	71.9
7	-1,0	0.2	1.8	1.4	2.2	-0.3	2.9	4.5	4.6	6.3	5.4
8	121	115	110	110	110	107	107.5	108	109	111	114
9	2910.42	2872.13	2882.11	2855.12	2867.58	2868.88	2905.13	2915.31	2999.07	3121.46	3138.64
10	119	112	107	104	102	98	96	93	92	93	91
11	8.82	8.38	8.3	8.34	8.31	8.64	8.64	8.63	8.61	8.68	8.68
12	0.81	0.81	0.8	0.82	0.82	0.82	0.81	0.83	0.83	0.83	0.84
13	80	79	80.0	79	78	79	81	81	81	80	80
14	82,00	81.80	80.20	80.33	80.52	80.77	81.20	82.18	82.52	82.79	83.02

Designations as in Table 2

Table 5. Value and number of PPP agreements concluded in a given year between 2009-2019 in the Netherlands and selected macroeconomic indicators

D	Years										
	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
W	0	0.94	0	0.86	1	0.79	0.54	0.78	**0.06**	**1.93**	0.85
L	0	2	0	2	1	3	2	4	1	2	1
1	624.84	639.19	650.36	652.97	660.46	671.56	690.01	708.34	738.15	773.99	810.25
2	356.96	433.17	479.24	510.1	505.65	506.34	514.31	515.93	577.09	615.6	633.06
3	32.34	34.05	29.04	26.27	19.86	15.26	14.5	-0.02	-9.65	-10.98	-14.39
4	354.95	378.94	401.48	432.59	447.08	455.87	446.26	438.66	420.3	405.77	394.67
5	8.9	70.5	130.9	174.3	202.6	322.6	337.6	433.7	441.7	556.2	729.2
6	1.4	11	20.1	26.7	30.7	48	48.9	61.2	59.8	71.9	90
7	9.7	2.6	8.1	6.2	9.8	4.4	0.2	3.8	4	5.6	0
8	252	265	277	282	282	289	287	289	277	267	255
9	793.55	805.38	825.96	848.86	865.20	873.03	883.21	892.51	907.92	959.75	988.51
10	127	126	127	130	131	130	128	126	123	124	122
11	9.53	8.99	8.99	9.0	8.84	8.92	8.92	8.8	8.89	8.89	9.01
12	0.87	0.88	0.82	0.9	0.86	0.86	0.83	0.86	0.85	0.85	0.84
13	89	88	89	84.0	83	83	87	83	82	82	82
14	16.49	16.57	16.66	16.7	16.78	16.83	16.9	16.98	17.08	17.18	17.28

Designations as in Table 2

Table 6. Value and number of PPP agreements concluded in a given year between 2009-2019 in Belgium and selected macroeconomic indicators

D	Years										
	2009	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
W	0.18	0.04	0.19	0.21	0	1.41	0.29	0	0.08	0	0.89
L	1	1	1	2	0	2	2	0	1	0	3
1	346.47	363.14	375.97	386.17	392.88	403	416.7	430.09	445.05	460.37	476.34
2	265.99	307.53	341.72	347.09	352.96	355.29	357.77	359.76	380.59	396.64	399.18
3	18.82	14.84	16.28	16.68	12.29	12.31	10.06	10.16	3.04	3.78	9.08
4	347.22	364.13	389.11	404.75	414.43	431.4	438.49	451.61	453.98	459.31	467.17
5	201.5	204.3	193.2	157.7	169.6	180.6	188.5	234.6	249.4	164.6	241
6	58.2	56.3	51.4	40.8	43.2	44.8	45.2	54.5	56	35.8	50.6
7	5.4	-0.7	22.2	14.6	8	-2	12	24	1.9	2.3	3.8
8	204.0	190.0	200.0	200	205	207	208.9	229.2	216.5	211.5	206.7
9	280.60	279.60	289.50	297.35	302.52	318.37	337.50	348.37	360.49	368.30	381.07
10	81.0	77.0	77.0	77	77	79	81.0	81	81	80	80
11	8.2	8.1	8.1	8.05	8.05	7.93	7.9	7.77	7.78	7.78	7.64
12	0.8	0.8	0.8	0.78	0.78	0.81	0.8	0.79	0.77	0.77	0.79
13	71.0	71.0	75.0	75	75	76	77.0	77	75	75	75
14	10.75	10.84	11.00	11.08	11.14	11.18	11.20	11.31	11.35	11.4	11.46

Designations as in Table 2

Table 7. Selected sets of explanatory variables used to build regression models

Number of set		**Number of explanatory variable**													
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
	w1	x	x	x	x	x	x	x		x		x		x	x
	w2	x		x	x	x	x	x	x	x	x		x	x	
	w3	x	x		x	x	x			x	x	x	x	x	x
Germany	w4	x	x	x	x	x	x	x	x			x	x	x	
France	w4	x	x	x	x	x	x	x		x	x	x			x
Great Britain	w4			x		x	x	x	x	x	x	x	x	x	x
Netherlands	w4	x	x	x	x	x		x	x		x	x	x		x
Belgium	w4		x	x	x	x		x	x	x	x	x	x	x	
	l1		x	x	x	x	x	x	x	x		x		x	x
	l2	x	x		x	x	x	x	x		x	x	x	x	
	l3	x		x		x	x		x	x	x	x	x	x	x
Germany	l4	x	x	x		x	x	x	x	x		x	x		x
France	l4	x	x	x	x	x	x	x	x	x		x	x		
Great Britain	l4	x	x	x	x	x	x	x		x		x	x	x	
Netherlands	l4	x	x			x	x	x	x	x		x	x	x	x
Belgium	l4		x	x	x	x		x	x	x	x	x	x	x	

Denotation of explanatory variables as in Table 2; w_i – for contract value models; l_i – for number of contracts

The indicators were selected on the basis of literature and own research. Available data cover the period 2009-2019. All 11 values of explanatory variables were used to determine the parameters of the model. The maximum number of explanatory (exogenous) variables in the models was assumed to be 8. Taking into account all variables leads to the need to consider 2^{14} models for each of the explanatory variables and each model type[1] (about 260 thousand).

[1] For the power and logarithmic models, less so due to the negativity of some of the explanatory variables.

In order to reduce the number of models considered, an initial selection of explanatory variables was made. For each explanatory variable, three sets of eleven explanatory variables were selected. For each set, all possible regression models were built with a number of explanatory variables from 3 to 8. Brute force method was applied. The criterion for selection of explanatory variables (for the first set) was the value of coefficients of linear correlation between the explained variable and explanatory variables (variables with the strongest correlation were selected). Then the remaining explanatory variables were introduced by eliminating variables that were strongly correlated with each other). On the basis of obtained results the fourth set[2] of eleven explanatory variables was selected, for which possible regression models were built, with the number of explanatory variables from 3 to 8 (Table 7). In case of models with one and two explanatory variables, all possible regression models were built. From among so built models, the best models were chosen according to the adopted criteria.

The models were chosen as criteria for compatibility (quality):

- R^2 coefficient:

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2} \tag{1}$$

- adjusted R^2 (R^2_{adj}):

$$R^2_{adj} = 1 - \frac{n-1}{n-p} \cdot \left(1 - R^2\right) \tag{2}$$

- mean absolute deviation (MAD):

$$MAD = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{n}\sum_{i=1}^{n} \left|y_i - \hat{y}_i\right| \tag{3}$$

- standard deviation of the absolute deviation:

$$SMAD = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d_i - \bar{x}_d)^2} \tag{4}$$

- standard error of the regression (SER):

$$SER = \sqrt{\frac{RSS}{n - p - 1}} \tag{5}$$

- Hannana-Quinna information criterion (HQC):

$$HQC = \ln\left(\frac{RSS}{n}\right) + \frac{2k \cdot \ln\left(\ln(n)\right)}{n} \tag{6}$$

- Akaike information criterion (AIC):

$$AIC = \ln\left(\frac{RSS}{n}\right) + \frac{2 \cdot (p + 1)}{n} \tag{7}$$

- corrected Akaike information criterion (AICc):

$$AICc = AICc + \frac{2k \cdot (k + 1)}{n \cdot (n - k - 1)} \tag{8}$$

- Bayesian information criterion (BIC):

$$BIC = \ln\left(\frac{RSS}{n}\right) + \frac{(p + 1) \cdot \ln(n)}{n} \tag{9}$$

Where: p - number of explanatory variables (predicators), k = p+1, $\bar{y}$ - mean value of the explained variable, y_i - actual value, $\hat{y}_i$ - model value (regression function), n - sample size.

Models considered were:

- no autocorrelation of residuals or Durbin-Watson test is inconclusive[3] (a significance level is assumed $\alpha = 0{,}05$),
- there is no multi-collinearity,
- explanatory variables are significant,
- R^2 coefficient is positive.

Two levels of non-multicollinearity (or is negligible) and significance of explanatory variables were assumed:

[2] Separately for each country and each explanatory variable.

[3] For 7 and 8 explanatory variables, an "inconclusive result" was assumed.

- level I (p1) – if at the same time:
  - for each explanatory variable, the variance inflation factor (VIF) is less than 10,
  - at the significance level $\alpha = 0.1$, reject the hypothesis that the parameter ai is equal to zero (t-test) and the p-value for the F-statistic is less than 0.1,
- level II (p2) – if at the same time:
  - for each explanatory variable, the VIF coefficient is less than 5,
  - at a significance level of $\alpha = 0.05$, reject the hypothesis that the parameter ai is equal to zero (t-test) and the p-value for the F-statistic is less than 0.05.

For nonlinear models, the significance of the explanatory variables and the F-statistic were verified for the linearized form.

In the second stage, the principal components were determined[4]:

- based on the covariance matrix,
- based on the matrix of linear correlation coefficients.

Treating the principal components as explanatory variables, all possible regression models were built (Table 1)[5]. The best models were selected. The criteria were applied as before[6]. For the sake of homogeneity 5 principal components were taken into account, which guarantees the inclusion of more than 97% of variability.

3. Results and analysis

Stage I

Linear regression models with one explanatory variable

In economics, the standard approach to assessing the power of the influence of a factor on the variable under study is to determine the slope coefficient of a simple regression. Tables 8 and 9 present the slope coefficients of simple regressions of the value and number of PPP contracts by country for all explanatory variables. A direct comparison of the slope coefficients to assess the power of the individual explanatory variables to influence the value and number of PPP contracts can lead to erroneous conclusions. This is due to the very large difference in the range of values of the individual explanatory variables. Therefore, the value of the increment of the explanatory variable is given when the value of the explanatory variable increases by 5% of the range of this variable (Table 8 and 9). Using these values, it can be concluded that the most powerful influence is exerted by the explanatory variables[7]:

- on the value of contracts:
  - France: 6(positive = p), 5(p), 11(negative = n), 14(n), 8(n), 4(n), 9(n), 1(n),
  - Great Britain: 6(n), 13(n), 5(n),
  - Germany: highest influence (positive) from variable 12,
  - Netherlands: 14(p), 11(n), 13(n), 12(p), 9(p), 1(p),
  - Belgium: highest influence (positive) is given by variable 12,
- on the number of contracts:
  - France: 6(p), 5(p), 8(n), 1(n), 9(n), 14(n), 3(p), 11(n),
  - Great Britain: 13(n), 6(n), 3(p), 8(p), 14(n), 7(n), 2(n),
  - Germany: 13(p), 11(p), 4(n), 12(p), 14(p), 8(p), 9(n),
  - Netherlands: 11(n), 4(p), 8(p),
  - Belgium: the highest influence (negative) is of variable 8 and of variable 12 (positive).

Note that in most cases, the significance of the explanatory variables in the regression equations is greater than 0.1 (Tables 8 and 9). In many cases there is a change in the sign of the slope of the regression straights. The change occurs both between countries (for the same explanatory variable) and between models of number of contracts and value of contracts (for the same country). The list of explanatory variables for which the slope are positive and negative is presented in Table 10. Also, in five cases there is autocorrelation of residuals[8].

The values of the coefficient of determination R^2 and the significance of the explanatory variables are also given. In the vast majority, the values of the R^2 coefficients are very low. They stand out:

- for explanatory variable 12 (Rule of Law Index) for the value of contracts for Germany and Belgium.
- for variable 6 (International Investment Position (% GDP)) for the number of contracts for France.

In these cases, the slope of the regression lines are significantly different from zero ($p < 0.05$)[9].

Taking all factors into account, one should be very cautious in assessing, on the basis of simple regressions, the power of the influence of individual explanatory variables on the value and number of PPP contracts in the countries under consideration.

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[4] Eigenvalues and eigenvectors were calculated using the website: https://www.dcode.fr/matrix-eigenvectors.

[5] Absolute value of the increment above €0.1 billion or number of contracts.

[6] Clearly this does not apply to logarithmic and power models.

[7] Absolute value of increment above EUR 0.1 billion or number of contracts.

[8] Negative for Germany for the value of contracts - explanatory variables 7, 9, 13, 14. Positive for France for the number of contracts – explanatory variable 11.

[9] For contract values, these are the only cases where the significance of the variables is less than 0.05.

Table 8. Slope and coefficients of determination R^2 of linear regression of PPP contract values

x	France				Great Britain				Germany				Netherlands				Belgium			
	R^2	a	b	ΔW	R^2	a	b	ΔW	R^2	a	b	ΔW	R^2	a	b	ΔW	R^2	a	b	ΔW
1	0.082	-0.004340	l	-0,10618	0.043	-0.001117	l	-0,05063	0.246	0.000705	?	0,03535	0.169	0.003894	l	0,13726	0.033	0.001969	l	0,01279
2	0.011	-0.005327	l	-0,04313	0.010	-0.003227	l	-0,02664	0.137	0.001076	?	0,02838	0.238	0.003501	?	0,09400	0.036	0.002225	l	0,01482
3	0.050	0.019107	l	0,08104	0.117	0.012103	l	0,07527	0.165	-0.003465	?	-0,03013	0.123	-0.010786	l	-0,00444	0.002	0.004248	l	0,00335
4	0.105	-0.002950	l	-0,11382	0.012	-0.000481	l	-0,02808	0.060	-0.000923	?	-0,02030	0.045	0.003745	l	0,07974	0.053	0.002573	l	0,01543
5	0.268	0.010248	l	0,19373	0.206	-0.003455	l	-0,12436	0.343	0.000408	?	0,03804	0.175	0.001068	l	0,01944	0.001	0.000458	l	0,00210
6	0.327	0.301078	l	0,21377	0.342	-0.112076	l	-0,15971	0.324	0.014705	?	0,03573	0.176	0.008633	l	0,02158	0.013	-0.006952	l	-0,00779
7	0.016	0.142037	l	0,04332	0.011	-0.041596	l	-0,02808	0.145	0.073569	n	0,02685	0.028	-0.027896	l	-0,00656	0.142	-0.019017	l	-0,02472
8	0.126	-0.048552	l	-0,11410	0.041	0.028628	l	0,04867	0.001	0.003539	?	0,00248	0.001	0.001080	l	0,01502	0.006	-0.003491	l	-0,00684
9	0.091	-0.004114	l	-0,11273	0.006	0.000861	l	0,01662	0.287	0.002455	n	0,03480	0.247	0.004633	l	0,20667	0.029	0.002066	l	0,01048
10	0.016	-0.354730	l	-0,03547	0.024	0.009938	l	0,03230	0.147	-0.019641	?	-0,02750	0.001	-0.004599	l	-0,02915	0.004	0.015216	l	0,00304
11	0.159	-7.461640	l	-0,13058	0.054	-2.466371	l	-0,04686	0.261	1.361373	?	0,03472	0.201	-1.289279	l	-0,57521	0.051	-0.618501	l	-0,01608
12	0.027	-15.575004	l	-0,05451	0.005	-10.350003	l	-0,01553	0.528	33.651771	l	0,05048	0.051	7.310236	l	0,31005	0.534	27.448712	l	0,05490
13	0.004	0.123600	l	0,02472	0.326	-0.357628	l	-0,14305	0.147	0.182547	n	0,02738	0.274	-0.101047	l	-0,42426	0.050	0.049955	l	0,01499
14	0.122	-0.978456	l	-0,11497	0.065	-0.260322	l	-0,06000	0.224	0.217278	n	0,03042	0.183	0.956148	l	0,80929	0.040	0.393089	l	0,01395

x – explanatory variable (according to Table 2), **R^2** – coefficient of determination, **a** – slope of the regression line, **b** – Durbin – Watson test result: l – no autocorrelation of residuals, n – negative autocorrelation, ? – inconclusive result, $\boldsymbol{\Delta W}$ = a(max(x) – min(x)) – the increase in the value of contracts with the increase in the independent variable by 5% of the variance. Significance of slope for explanatory variables:
- $p < 0.05$: Germany – 12, Belgium – 12,
- $p < 0.1$: France – 6, Great Britain – 6, Germany – 5, 6, 9,12, Netherlands – 13, Belgium – 12,
- in other cases $p > 0.1$.

Table 9. Slope and coefficients of determination R^2 of linear regression of PPP contract numbers

x	France				Great Britain				Germany				Netherlands				Belgium			
	R^2	a	b	ΔL	R^2	a	b	ΔL	R^2	a	b	ΔL	R^2	a	b	ΔL	R^2	a	b	ΔL
1	0.171	-0.005759	l	-0,14090	0.176	-0.002156	l	-0,09773	0.009	-0.000291	?	-0,04420	0.020	0.002850	l	0,02642	0.020	0.003416	l	0,02218
2	0.035	-0.008886	?	-0,07194	0.165	-0.012360	l	-0,10203	0.049	-0.001423	l	-0,08415	0.047	0.003337	l	0,04606	0.015	0.003177	l	0,02115
3	0.128	0.028005	l	0,11878	0.308	0.018767	l	0,11672	0.001	-0.000442	l	0,00049	0.060	-0.016262	l	-0,03939	0.025	0.030512	l	0,02407
4	0.176	-0.003493	l	-0,13475	0.136	-0.001525	l	-0,08897	0.178	-0.003489	?	-0,37593	0.335	0.021921	?	0,11061	0.012	0.002703	l	0,01621
5	0.368	0.011002	l	0,20799	0.105	-0.002353	l	-0,08469	0.000	0.000019	l	0,00147	0.086	0.001617	l	0,05825	0.019	0.004342	l	0,01991
6	0.417	0.311766	l	0,22135	0.249	-0.091176	l	-0,12993	0.002	0.002635	?	0,00641	0.128	0.015879	?	0,07034	0.000	0.001691	l	0,00189
7	0.049	-0.228440	l	-0,06967	0.154	-0.151506	l	-0,10227	0.017	-0.056035	?	-0,00878	0.189	-0.155765	l	-0,07632	0.046	-0.023841	l	-0,03099
8	0.267	-0.064890	l	-0,15249	0.211	0.062191	l	0,10572	0.017	0.032267	?	0,17709	0.350	0.054152	?	0,10018	0.097	-0.030521	l	-0,05982
9	0.164	-0.005059	l	-0,13863	0.062	0.002718	l	0,05247	0.006	-0.000801	?	-0,11806	0.051	0.004543	l	0,04428	0.018	0.003555	l	0,01803
10	0.016	0.324324	?	0,03243	0.164	0.024726	l	0,08036	0.012	0.012557	?	0,06132	0.019	0.056962	l	0,02563	0.001	0.018919	l	0,00378
11	0.146	-6.547619	p	-0,11458	0.180	-4.291441	?	-0,08154	0.280	3.097658	?	1,32304	0.327	-3.544363	?	-0,12937	0.027	-0.991220	l	-0,02577
12	0.031	15.312496	?	0,05359	0.001	-4.166669	l	-0,00625	0.008	8.928556	l	0,36842	0.085	20.180718	l	0,06054	0.087	24.358969	?	0,04872
13	0.037	-0.346667	?	-0,06933	0.363	-0.360000	?	-0,14400	0.137	0.386792	?	1,54488	0.186	-0.179266	l	-0,06274	0.006	0.038462	l	0,01154
14	0.174	-1.072032	l	-0,12596	0.214	-0.451633	?	-0,10410	0.008	0.088077	?	0,35897	0.070	1.268807	l	0,05012	0.013	0.488124	l	0,01733

x – explanatory variable (according to Table 2), $\mathbf{R^2}$ – coefficient of determination, **a** – slope of the regression line, **b** – Durbin – Watson test result: l – no autocorrelation of residuals, n – negative autocorrelation, ? – inconclusive result, ΔL = a(max(x) – min(x)) – the increase in the value of contracts with the increase in the independent variable by 5% of the variance.
Significance of slope for explanatory variables:
- $p < 0.05$: France – 5, 6, Great Britain – 13, Belgium – 12,
- $p < 0.1$: France – 5, 6, Great Britain – 3, 13, Germany – 11, 6, 9, 12, Netherlands – 8, 11,
- in other cases $p > 0.1$.

Table 10. List of explanatory variables for which the slope of regression lines are positive and negative

Country	value of contracts		number of contracts	
	positive	negative	positive	negative
France	3, 5, 6, 7, 13	1, 2, 4, 8, 9, 10, 11, 12, 14	3, 5, 6, 10, 12	1, 2, 4, 7, 8, 9, 11, 13, 14
Great Britain	3, 8, 9, 10	1, 2, 4, 5, 6, 7, 11, 12, 13, 14	3, 8, 9, 10	1, 2, 4, 5, 6, 7, 11, 12, 13, 14
Germany	1, 2, 5, 6, 7, 8, 9, 11, 12, 13, 14	3, 4, 10	5, 6, 8, 10, 11, 12, 13, 14	1, 2, 3, 4, 7, 9
Netherlands	1, 2, 4, 5, 6, 8, 9, 12, 14	3, 7, 10, 11, 13	1, 2, 4, 5, 6, 8, 9, 10, 12, 14	3, 7, 11, 13
Belgium	1, 2, 3, 4, 5, 9, 10, 12, 13, 14	6, 7, 8, 11	1, 2, 3, 4, 5, 6, 9, 10, 12, 13, 14	7, 8, 11

Designation of explanatory variables as in Table 2.

Best regression models for value and number of PPP contracts

France. For contract values, 22 models satisfying the p1 criterion and 15 models satisfying the p2 criterion were obtained (Table 21). These models are (Table 11):
- 9 models with one explanatory variable – linear, logarithmic and hyperbolic I models with variables 7, 10, 12,
- 9 models with two variables[10] with variables (10, 11), (10, 13), (11, 12),
- 2 log-hyperbolic models with variables (5, 10, 14) and (6, 10, 14),
- log-hyperbolic model with variables (5, 10, 11, 14), (6, 10, 11, 14).

The best models are shown in Table 11[11]. The three best models (considering all criteria) are the linear, logarithmic and hyperbolic I models with variables 11 and 12. These models also satisfy the p2 criterion. Not all criteria position the models in the same way. According to the criteria R^2, R^2_{adj}, and MAD the best model is the log-hyperbolic model with explanatory variables 5, 10 and 14 (according to the other criteria it ranks fourth or further)[12]. Note that the linear model with variable 12 is ranked fifth (considering all criteria). Among models fulfilling criterion p1 variables 1, 2, 3, 4, 8 and 9 do not occur (Table 21). Among the models satisfying criterion p2, variables 6, 7 and 13 are additionally present.

For the number of PPP contracts, 33 models satisfying the p1 criterion were obtained, including 9 satisfying the p2 criterion. These are:
- 18 models with three variables (15 hyperbolic I, 2 logarithmic and 1 linear),
- 11 models with two variables (5 hyperbolic I, 3 logarithmic and 3 linear – based on variable 2 + one of the variables, 4, 8, 14, 3, 5),
- 4 models with one variable (2 linear and 2 hyperbolic I with variables 5[13] and 6).

The best models are shown in Table 12. All criteria, except the SMAD criterion, indicate the same five models as the best[14]. The SMAD criterion as the best indicates the model L_F6 (hyperbolic I with variables 1, 5 and 7) positioned by the other criteria in seventh position. The best models (hyperbolic I, logarithmic and linear) use variables 2, 8 and 13. The best model satisfying criterion p2 is hyperbolic I (L_F4) with variables 2, 5, 7. This model is positioned fourth by all criteria (among models satisfying criterion p1). All variables enter at least once in models satisfying criterion p1 (Table 21). Variable 2 appears in 21 models (including the top five models). However, only variables 2, 5, 6, 7 and 8 occur among models satisfying criterion p2.

Great Britain. For the value of contracts, 24 models were obtained, 6 of which satisfied the p2 criterion. These are:
- 2 hyperbolic I models with six variables,
- 1 linear model with five variables,
- 11 linear models with four variables,
- 6 linear models with three variables,
- 4 linear models with one variable (linear model with variable 6 and linear, logarithmic and hyperbolic I models with variable 13).

[10] Hyperbolic II and hyperbolic III models are not present.

[11] Tables with the best models were created in such a way that there were the best models according to each criterion satisfying condition p1 and the best models (according to all criteria combined) satisfying criterion p2 (if any).

[12] The other criteria are in line with the models indicated earlier.

[13] Except in one case (for variable 5), variables 5 and 6 do not occur in models with two explanatory variables. They occur in models with three variables (but not simultaneously).

[14] For the L_F5 model positioned fifth, only the MAD criterion indicates a position of sixth.

The best model is shown in Table 13. The best model according to all criteria except the AICc criterion is the linear model with variables 3, 6, 8, 9 ($W_{GB}1$). According to the AICc criterion, the $W_{GB}1$ model ranks second. The best, according to the AICc criterion, is the linear model with variables 3, 5, 9 ($W_{GB}2$). This model, according to six of the other eight criteria, is positioned in second place. This model satisfies criterion p2. The third model is a linear model with variables 3, 6, 9 ($W_{GB}3$). Note that there is a very strong linear relationship between variables 5 and 6 ($\rho = 0.967$). However, the $W_{GB}3$ model is ranked lower than the $W_{GB}2$ model. Moreover, it does not satisfy the p2 criterion. Among the models that satisfy the p2 criterion, the linear model with variables 3, 6, 7, 12 ($W_{GB}5$) should be considered the second best (considering all criteria). The criteria place this model in positions from fourth to thirteenth (considering all models). Among the models satisfying criterion p1, there are no models using variable 10 (Table 21). Variables 6 and 7 (12 times each) and 2 and 9 (9 times each) appear the highest number of times in the models. However, variables 6 and 9 occur in the top five models (4 times each). For models satisfying the p2 criterion, there are no models using variables 1, 2, 4, 8, 10 and 13.

For the number of PPP contracts, 45 models were obtained, of which 6 satisfied the p2 criterion. These are:

- 2 linear models with six variables,
- 6 linear models with five variables,
- 10 models with four variables (8 linear models and 2 hyperbolic I models),
- 12 models with three variables (10 linear models and 2 hyperbolic I models),
- 11 models with two variables (10 linear models and 1 hyperbolic I model),
- 4 linear models with one variable (linear model with variable 3 and linear, logarithmic and hyperbolic I models with variable 13).

The best models are shown in Table 14. Eight of the nine criteria (except AICc) indicate the same models as the best:

1. linear with variables 2, 3, 6, 7, 12, 13 ($L_{GB}1$),
2. linear with variables 2, 3, 4, 6, 9, 12 ($L_{GB}2$),
3. linear with variables 2, 3, 6, 7, 12 ($L_{GB}2$)[15].

According to the AICc criterion, these models are in distant positions – 27, 44 and 6 respectively. The best according to this criterion is a linear model with variables 4, 5, 9, 12 ($L_{GB}4$). Other criteria place this model in fourth position (5 criteria), fifth (2 criteria) and sixth (SMAD criterion). None of these models satisfy the p2 criterion. The best model satisfying the p2 criterion is the linear model with variables 3, 6, 12. The criteria place this model in positions in the second ten. Only the AICc criterion in the fifth position. In models satisfying criterion p1 all explanatory variables are used (Table 21). The most frequent variable is 6 (in 21 models). However, in the best models it does not occur. Variables 3, 6 and 12 are present in the three best models – also in the best model satisfying criterion p2. Only variables 2, 3, 6 (3 times), 12 and 13 (3 times) are used in the models meeting criterion p2.

Table 11. Best models of PPP contract values in France between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
W_F1	$4.802x_{11}+51.340x_{12}-74.907$	0.817 (2)	0.771 (2)	0.536 (2)	0.374 (1)	0.755 (1)	-0.402 (1)	-0.402 (1)	-0.022 (1)	-0.225 (1)	p2
W_F2	$38.403\ln x_{11}+38.704\ln x_{12}-66.686$	0.814 (3)	0.768 (3)	0.539 (3)	0.379 (2)	0.761 (2)	-0.387 (2)	-0.387 (2)	-0.007 (2)	-0.210 (2)	p2
W_F3	$-\frac{307.094}{x_{11}}-\frac{9.157}{x_{12}}+79.326$	0.812 (4)	0.764 (4)	0.542 (4)	0.384 (3)	0.767 (3)	-0.372 (3)	-0.372 (3)	0.008 (3)	-0.195 (3)	p2
W_F4	$\exp\left(-\frac{5311.97}{x_5}-\frac{44853.75}{x_{10}}-\frac{21651.31}{x_{14}}+792.018\right)$	0.898 (1)	0.855 (1)	0.461 (1)	0.554 (10)	0.878 (4)	-0.076 (4)	-0.076 (4)	0.622 (10)	0.160 (7)	p1
W_F5	$53.981x_{12}-38.900$	0.679 (6)	0.643 (6)	0.729 (12)	0.465 (6)	0.943 (5)	0.001 (5)	0.047 (5)	0.183 (4)	0.119 (4)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

[15] For this model, the SMAD criterion indicates position four.

Table 12. Best models of the number of PPP contracts in France between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
L_F1	$-\frac{14507,78}{x_2}+\frac{8124,65}{x_8}+\frac{4427,45}{x_{13}}-71,652$	0.874 (1)	0.821 (1)	0.517 (1)	0.998 (8)	0.894 (1)	0.596 (1)	0.051 (1)	0.657 (1)	0.196 (1)	p1
L_F2	$0,092\ln x_2-0,268\ln x_8-67,219\ln x_{13}+308,116$	0.864 (2)	0.806 (2)	0.560 (2)	0.911 (6)	0.930 (2)	0.676 (2)	0.131 (2)	0.737 (2)	0.275 (2)	p1
L_F3	$36,884x_2-46,838x_8-1,006x_{13}-81,989$	0.847 (3)	0.781 (3)	0.610 (3)	0.857 (10)	0.987 (3)	0.795 (3)	0.250 (3)	0.856 (3)	0.395 (3)	p1
L_F4	$-\frac{5957,27}{x_2}-\frac{1930,46}{x_5}+\frac{8,755}{x_7}+7,714$	0.843 (4)	0.775 (4)	0.649 (4)	0.753 (4)	1.001 (4)	0.822 (4)	0.277 (4)	0.883 (4)	0.422 (4)	p2
L_F5	$-\frac{5651,73}{x_2}-\frac{2120,35}{x_5}+\frac{10,230}{x_8}+5.849$	0.837 (5)	0.767 (5)	0.665 (6)	0.737 (5)	1.018 (5)	0.856 (5)	0.312 (5)	0.918 (5)	0.456 (5)	p1
L_F6	$-\frac{67871,9}{x_1}-\frac{2576,53}{x_5}+\frac{12,668}{x_7}+22,500$	0.821 (7)	0.745 (7)	0.722 (10)	0.652 (1)	1.066 (7)	0.948 (7)	0.404 (7)	1.010 (7)	0.548 (7)	p1

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Table 13. Best value models of PPP contracts in Great Britain between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
$W_{GB}1$	$0.058x_3-0.185x_6-0.089x_8-0.011x_9+43.808$	0.921 (1)	0.868 (1)	0.352 (1)	0.248 (1)	0.574 (1)	-0.920 (1)	-0.806 (1)	0.285 (2)	-0.625 (1)	p1
$W_{GB}2$	$0.049x_3-0.008x_5-0.013x_9+34.936$	0.886 (4)	0.837 (2)	0.442 (6)	0.264 (2)	0.638 (2)	-0.716 (2)	-0.625 (2)	-0.019 (1)	-0.480 (2)	p2
$W_{GB}3$	$0.036x_3-0.182x_6-0.010x_9+28.636$	0.837 (8)	0.768 (3)	0.487 (10)	0.381 (8)	0.762 (3)	-0.360 (3)	-0.269 (3)	0.337 (3)	-0.124 (3)	p1
$W_{GB}4$	$-\frac{51335.6}{x_1}+\frac{3267.58}{x_2}+\frac{8.31}{x_6}+\frac{180138.5}{x_9}+\frac{1211.63}{x_{11}}+\frac{4008.61}{x_{13}}-234.47$	0.889 (2)	0.722 (6)	0.432 (3)	0.833 (6)	0.833 (6)	-0.264 (4)	-0.105 (4)	0.337 (24)	0.148 (7)	p1
$W_{GB}5$	$0.038x_3-0.190x_6+0.277x_7+84.108x_{12}-74.131$	0.839 (6)	0.839 (4)	0.529 (13)	0.818 (4)	0.818 (4)	-0.213 (6)	-0.099 (5)	0.991 (9)	0.081 (5)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Table 14. Best models for the number of PPP contracts in Great Britain between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
$L_{GB}1$	$-0.027x_2+0.035x_3-0.231x_6+0.403x_7+64.800x_{12}+0.159x_{13}-61.346$	0.989 (1)	0.971 (1)	0.116 (1)	0.106 (1)	0.255 (1)	-2.630 (1)	-2.471 (1)	0.923 (27)	-2.217 (1)	p1
$L_{GB}2$	$-0.015x_2+0.042x_3+0.003x_4-0.174x_6-0.006x_9+49.574x_{12}-23.914$	0.974 (2)	0.934 (2)	0.201 (2)	0.125 (2)	0.388 (2)	-1.792 (2)	-1.633 (2)	1.761 (44)	-1.379 (2)	p1
$L_{GB}3$	$-0.026x_2+0.028x_3-0.200x_6+0.367x_7+61.475x_{12}-45.474$	0.963 (3)	0.927 (3)	0.232 (3)	0.156 (4)	0.408 (3)	-1.625 (3)	-1.488 (3)	0.421 (6)	-1.271 (3)	p1
$L_{GB}4$	$0.048x_3-0.008x_5-0.008x_9+58.58x_{12}-28.112$	0.949 (5)	0.915 (4)	0.270 (5)	0.191 (6)	0.441 (4)	-1.450 (4)	-1.336 (4)	-0.24 (1)	-1.155 (4)	p1
$L_{GB}5$	$0.026x_3-0.151x_6+82.458x_{12}-70.333$	0.821 (17)	0.745 (14)	0.541 (18)	0.289 (11)	0.761 (14)	-0.361 (14)	-0.270 (13)	0.336 (5)	-0.125 (13)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Germany. For contract values, 53 models were obtained, of which 14 satisfy the p2 criterion. These models are:

- linear with variables 5, 9, 10, 12, 13,
- 20 models with four variables (7 logarithmic models, 7 linear models, 6 hyperbolic I models),
- 20 models with three variables (7 log-linear models, 8 linear models, 5 hyperbolic I models),
- 3 models with two variables (linear, logarithmic and hyperbolic I models with variables 11 and 12),
- 9 models with one variable (logarithmic, linear and hyperbolic I models with variables 9 and 12, power and linear models with variable 6 and linear model with variable 5).

The best models (satisfying criterion p1 and p2) are presented in Table 15. In the models of contract values, there are greater discrepancies in the assessment of model fit by different criteria in relation to models for France and the GB. However, the best (considering all criteria) are models with four explanatory variables using variables 2, 4, 11, 12. These are the logarithmic, linear and hyperbolic I models. The hyperbolic I model with variables 4, 10, 11, 12 should be indicated as the fourth one. The AICc criterion best indicates the linear model with variables 7, 11, 12 (W_D6). However, the others place this model in distant positions (4 criteria in position 11, MAD in position 13, R^2 and R^2_{adj} at position 14, SMAD at position 30). At the same time, it is the best model satisfying the p2 criterion. The second-best model satisfying the p2 criterion is the hyperbolic model I with variables 1, 8, 11, 12 (W_D7)[16]. Models satisfying criterion p1 do not use variables 3 and 8 (Table 21). As many as 47 models use variable 12 (also in the best models). Models satisfying criterion p2 do not use variables 1, 3, 4, 8, 9 and 12. Variable 12 is present in all models.

For the number of contracts, 23 models were obtained, 6 of which satisfy the p2 criterion. These models are:

- 11 models with four variables (4 logarithmic models, 7 linear models, 4 hyperbolic I models),
- 5 models with three variables (logarithmic, linear and hyperbolic I models based on variables 9, 11, 12, a linear model with variables 1, 11, 12 and a hyperbolic I model with variables 2, 7, 13),
- 3 models with two variables (linear, logarithmic and hyperbolic models with variables 4 and 8),
- 4 models with one variable (logarithmic, linear and hyperbolic I models with variable 11 and, linear model with variable 7).

The best models are shown in Table 16. All criteria indicate quite clearly the three best models. These are the hyperbolic I (L_D1), logarithmic (L_D2) and linear (L_D3) models with variables 2, 8, 11 and 12. There are differences for the SMAD criterion (L_D1 fourth position, L_D3 first position) and AICc (L_D1 third position, L_D2 fifth position, L_D3 eighth position). These models satisfy the p2 criterion. According to the AICc criterion, the best model is the linear model with variables 9, 11 and 12 (L_D4). This model does not satisfy criterion p2. Models satisfying the p1 criterion do not use variables 3 and 14 (Table 21). The most frequently used variables are 11, 12 (found in all the best models) and variable 8. Models satisfying criterion p2 use variables 8, 11, 12, 2, 1 and 4.

Netherlands. For contract values, 15 models were obtained. There are no models that satisfy the p2 criterion. These models are:

- logarithmic model with variables 2, 5, 7, 8 and 10,
- 9 models with four variables (2 hyperbolic I, 4 logarithmic and 3 linear),
- 3 models with two variables (linear, hyperbolic I and logarithmic models with variables 2 and 12),
- linear and logarithmic model with variable 13.

The best models are presented in Table 17. All criteria except AICc indicate as the best the logarithmic model with variables 2, 5, 7, 8 and 10 (W_N1) and the hyperbolic I model with variables 1, 4, 10 and 11 (W_N2). The AICc criterion places these models in 15th and 6th position, respectively. The hyperbolic model I with variables 4, 9, 10 and 11 ranks third according to the seven criteria (fifth according to the MAD criterion, seventh according to the AICc criterion). The best model according to the AICc criterion is a linear model with variable 13. The AICc criterion, as in the case of the number of PPP agreements in the GB, values the obtained models in a significantly different way than the other criteria. The most frequent variables in the models are 10 and 4 (Table 21).

For the number of contracts, 48 models were obtained, of which 5 satisfied the p2 criterion. These models are:

- 17 models with four variables (4 logarithmic models, 9 linear models, 4 hyperbolic I models),
- 17 models with four variables (6 logarithmic models, 8 linear models, 3 hyperbolic I models),
- 4 models with two variables (2 hyperbolic I models and 2 log-linear models with the variables (6, 12) and (5, 12)[17],
- 10 models with one variable (3 linear, 3 hyperbolic I and 3 logarithmic with variables 4, 8 and 11 and hyperbolic I with variable 3).

[16] It ranks highest (18th) according to the R^2 and MAD criteria, and lowest (42nd) according to the AICc criterion.

[17] The linear correlation coefficient between variables 5 and 6 is 0.995.

The best models are shown in Table 18. The best 3 models (according to all criteria[18]) are linear models with variables:

1. 5, 8, 12 and 13 (L_N1),
2. 6, 8, 12, 13 (L_N2),
3. 8, 9, 12, 13 (L_N3).

The best models satisfying the p2 criterion are linear models with variables:

1. 6, 8 in 12 (L_N4),
2. 5, 8 in 12 (L_N5),
3. 7, 8 in 12 (L_N6).

These models are ranked in distant positions (including models with criterion p1). The highest positions (7th, 9th and 10th respectively) are occupied by the AICc criterion. Variable 12 is found in 36 models satisfying criterion p1 and 4 models satisfying criterion p2, variable 8 in 27 models satisfying criterion p1 and in three satisfying criterion p2 (Table 21). Both variables occur in the best models. There are no models with variable 13 among the models satisfying criterion p1. The models satisfying criterion p2 use variables 3, 5, 6, 7, 8 and 12.

Table 15. Best models of PPP contract values in Germany between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
W_D1	$-3.397\ln x_2+5.458\ln x_4+22.263\ln x_{11}+50.401\ln x_{12}-55.365$	0.924 (1)	0.873 (1)	0.102 (5)	0.072 (2)	0.166 (1)	-3.400 (1)	-3.286 (1)	-2.195 (2)	-3.105 (1)	p1
W_D2	$\frac{3893.46}{x_2}-\frac{13101.5}{x_4}-\frac{193.06}{x_{11}}-\frac{41.92}{x_{12}}+76.74$	0.924 (2)	0.873 (2)	0.101 (4)	0.073 (5)	0.166 (2)	-3.399 (2)	-3.285 (2)	-2.194 (3)	-3.104 (2)	p1
W_D3	$-0.003x_2+0.002x_4+2.551x_{11}+60.549x_{12}-72.560$	0.923 (3)	0.872 (3)	0.102 (6)	0.073 (4)	0.167 (3)	-3.387 (3)	-3.273 (3)	-2.182 (4)	-3.092 (3)	p1
W_D4	$-\frac{14171.4}{x_4}-\frac{713.30}{x_{10}}-\frac{263.59}{x_{11}}-\frac{42.20}{x_{12}}+96.490$	0.917 (4)	0.860 (6)	0.109 (11)	0.072 (1)	0.175 (6)	-3.302 (6)	-3.188 (6)	-2.097 (14)	-3.007 (6)	p1
W_D5	$-0.003x_2+0.022x_6+47.481x_{12}+0.153x_{13}-47.809$	0.911 (8)	0.852 (8)	0.092 (1)	0.100 (18)	0.180 (8)	-3.243 (8)	-3.129 (8)	-2.038 (17)	-2.948 (8)	p1
W_D6	$-0.105x_7+1.442x_{11}+50.800x_{12}-53.346$	0.866 (14)	0.793 (14)	0.120 (13)	0.115 (30)	0.204 (11)	-2.992 (11)	-2.901 (11)	-2.295 (1)	-2.756 (11)	p2
W_D7	$-\frac{1722.78}{x_5}-\frac{1258.48}{x_{10}}-\frac{38.46}{x_{12}}-\frac{1464.07}{x_{13}}+79.823$	0.848 (18)	0.746 (23)	0.140 (18)	0.108 (25)	0.235 (23)	-2.704 (24)	-2.590 (26)	-1.499 (42)	-2.409 (26)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied

Table 16. Best models for the number of PPP contracts in Germany 2009-2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
L_D1	$\frac{12081.67}{x_2}+\frac{3354.23}{x_8}-\frac{335.68}{x_{11}}-\frac{66.75}{x_{12}}+80.657$	0.856 (1)	0.760 (1)	0.297 (1)	0.233 (4)	0.503 (1)	-1.185 (1)	-1.071 (1)	0.020 (3)	-0.890 (1)	p2
L_D2	$-11.386\ln x_2-27.304\ln x_8+44.527\ln x_{11}+81.905\ln x_{12}+130.350$	0.855 (2)	0.759 (2)	0.308 (2)	0.220 (2)	0.505 (2)	-1.179 (2)	-1.065 (2)	0.026 (5)	-0.884 (2)	p2
L_D3	$-0.010x_2-0.218x_8+5.732x_{11}+99.961x_{12}-93.855$	0.854 (3)	0.757 (3)	0.316 (3)	0.210 (1)	0.507 (3)	-1.171 (3)	-1.057 (3)	0.034 (8)	-0.876 (3)	p2
L_D4	$-0.012x_9+6.287x_{11}+80.407x_{12}-84.148$	0.730 (7)	0.614 (7)	0.459 (11)	0.231 (3)	0.638 (7)	-0.715 (7)	-0.623 (7)	-0.017 (1)	-0.479 (7)	p1
L_D5	$-14.354\ln x_1-19.392\ln x_8+59.644\ln x_{11}+82.215\ln x_{12}+95.342$	0.790 (5)	0.650 (5)	0.336 (5)	0.312 (9)	0.645 (8)	-0.809 (5)	-0.695 (5)	0.396 (18)	-0.514 (5)	p2
L_D6	$\frac{41874.1}{x_1}+\frac{2381.72}{x_8}-\frac{495.34}{x_{11}}-\frac{66.61}{x_{12}}+104.458$	0.784 (6)	0.641 (6)	0.343 (6)	0.313 (10)	0.652 (9)	-0.781 (6)	-0.667 (6)	0.424 (19)	-0.486 (6)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

[18] Only for the LH3 does the AICc criterion place this model in fifth position.

Table 17. Best models of PPP contract values in the Netherlands between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
W_N1	$-8.785\ln x_2+2.249\ln x_5+0.143\ln x_7-33.158\ln x_8+31.448\ln x_{10}+77.386$	0.833 (1)	0.666 (1)	0.193 (1)	0.107 (1)	0.324 (1)	-2.090 (1)	-1.953 (1)	-0.044 (15)	-1.736 (1)	p1
W_N2	$-\frac{8617.92}{x_1}+\frac{5780.6}{x_4}-\frac{6549.20}{x_{10}}+\frac{297.90}{x_{11}}+17.766$	0.770 (2)	0.617 (2)	0.216 (2)	0.144 (2)	0.346 (2)	-1.931 (2)	-1.817 (2)	-0.726 (6)	-1.636 (2)	p1
W_N3	$\frac{5676.73}{x_4}-\frac{10457.6}{x_9}-\frac{4877.48}{x_{10}}+\frac{298.24}{x_{11}}+4.240$	0.760 (3)	0.600 (3)	0.222 (5)	0.145 (3)	0.354 (3)	-1.887 (3)	-1.773 (3)	-0.682 (7)	-1.592 (3)	p1
W_N4	$2.555\ln x_2+15.085\ln x_{12}-15.085$	0.475 (13)	0.344 (13)	0.248 (12)	0.308 (12)	0.451 (11)	-1.435 (11)	-1.366 (11)	-1.054 (3)	-1.258 (11)	p1
W_N5	$-0.101x_{13}+9.266$	0.274 (14)	0.193 (14)	0.324 (14)	0.335 (15)	0.503 (14)	-1.256 (14)	-1.211 (14)	-1.074 (1)	-1.138 (14)	p1
W_N6	$-113.359\ln x_{13}+500.347$	0.272 (15)	0.191 (15)	0.326 (15)	0.334 (14)	0.504 (15)	-1.254 (15)	-1.208 (15)	-1.072 (2)	-1.136 (15)	p1

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Table 18. Best models for the number of PPP contracts in the Netherlands between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
L_N1	$0.008x_5+0.095x_8+72.118x_{12}+0.397x_{13}-122.137$	0.924 (1)	0.873 (1)	0.258 (1)	0.195 (1)	0.430 (1)	-1.500 (1)	-1.386 (1)	-0.295 (1)	-1.205 (1)	p1
L_N2	$0.062x_6+0.081x_8+70.181x_{12}+0.386x_{13}-115.868$	0.919 (2)	0.865 (2)	0.260 (2)	0.208 (2)	0.443 (2)	-1.437 (2)	-1.323 (2)	-0.232 (2)	-1.142 (2)	p1
L_N3	$0.110x_8+0.035x_9+83.692x_{12}+0.535x_{13}-175.769$	0.897 (3)	0.828 (3)	0.298 (3)	0.228 (3)	0.500 (3)	-1.199 (3)	-1.085 (3)	0.006 (5)	-0.904 (3)	p1
L_N4	$0.023x_6+0.061x_8+42.388x_{12}-52.217$	0.760 (18)	0.657 (16)	0.400 (12)	0.416 (22)	0.706 (16)	-0.511 (16)	-0.420 (15)	0.186 (7)	-0.275 (15)	p2
L_N5	$0.003x_5+0.066x_8+42.763x_{12}-53.806$	0.759 (19)	0.656 (17)	0.403 (13)	0.414 (21)	0.707 (17)	-0.509 (17)	-0.418 (16)	0.188 (9)	-0.273 (16)	p2
L_N6	$-0.180x_7+0.060x_8+35.890x_{12}-44.681$	0.758 (20)	0.654 (18)	0.427 (16)	0.389 (19)	0.709 (18)	-0.504 (18)	-0.413 (17)	0.194 (10)	-0.268 (17)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Belgium. For the value of contracts, 20 models satisfying the p1 criterion were obtained, including 14 satisfying the p2 criterion. These models are:

- 3 hyperbolic I models with six variables,
- 5 models with five variables (linear and logarithmic models and 3 hyperbolic I models),
- 4 models with four variables (hyperbolic I, logarithmic and linear models with variables 8, 10, 12, 13 and a linear model with variables 3, 7, 11, 13),
- 5 linear models with three variables,
- 3 models with variable 12 (logarithmic, linear, hyperbolic I).

The best models are shown in Table 19. The four best models, according to all criteria except the AICc criterion, are the hyperbolic I model with variables:

1. 3, 8, 9, 11, 12, 13 (W_B1),
2. 2, 7, 8, 10, 11, 12 (W_B2),
3. 4, 7, 8, 10, 11, 12 (W_B3),
4. 7, 8, 10, 12, 13 (W_B4).

The AICc criterion positions these models fourth, fifth, twentieth and seventh respectively. The AICc criterion identifies as the best the linear, logarithmic and hyperbolic I models with variable 12. These models and the W_B4 model satisfy the p2 criterion. Model W_B4 is the best of the models satisfying criterion p2. Models satisfying criterion p1 do not use variables 1, 5, 6 and 9 (Table 21). The most frequent variable is 12 (in all the best models). Models satisfying criterion p1 also do not use variables 1, 5, 6 and 9. Variables 8 and 12 are the most frequent.

Table 19. Best models of PPP contract values in Belgium between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
W_B1	$-\frac{0.166}{x_3}+\frac{1039.93}{x_8}-\frac{2958.95}{x_9}-\frac{281.08}{x_{11}}-\frac{24.33}{x_{12}}-\frac{6.07}{x_{13}}+71.178$	0.992 (1)	0.981 (1)	0.026 (1)	0.017 (1)	0.062 (1)	-5.469 (1)	-5.310 (1)	-1.916 (4)	-5.057 (1)	p1
W_B2	$-\frac{1412.76}{x_2}+\frac{0.444}{x_7}+\frac{2085.90}{x_8}-\frac{1789.74}{x_{10}}-\frac{141.89}{x_{11}}-\frac{29.43}{x_{12}}+72.421$	0.992 (2)	0.980 (2)	0.033 (2)	0.029 (2)	0.063 (2)	-5.424 (2)	-5.264 (2)	-1.870 (5)	-5.011 (2)	p1
W_B3	$-\frac{1977.45}{x_4}+\frac{0.451}{x_7}+\frac{2282.09}{x_8}-\frac{1215.57}{x_{10}}-\frac{106.56}{x_{11}}-\frac{26.65}{x_{12}}+56.877$	0.973 (3)	0.933 (3)	0.056 (3)	0.044 (3)	0.116 (3)	-4.214 (3)	-4.054 (3)	-0.660 (20)	-3.801 (3)	p1
W_B4	$\frac{0.473}{x_7}+\frac{2439.94}{x_8}-\frac{1191.39}{x_{10}}-\frac{25.03}{x_{12}}-\frac{766.79}{x_{13}}+45.814$	0.955 (4)	0.910 (4)	0.076 (4)	0.052 (4)	0.134 (4)	-3.850 (4)	-3.713 (4)	-1.804 (7)	-3.496 (4)	p2
W_B5	$-\frac{548.30}{x_2}+\frac{0.488}{x_7}+\frac{1880.04}{x_8}-\frac{1030.01}{x_{10}}-\frac{26.28}{x_{12}}+39.389$	0.906 (8)	0.812 (8)	0.105 (7)	0.081 (9)	0.194 (8)	-3.114 (8)	-2.978 (8)	-1.069 (14)	-2.761 (9)	p2
W_B6	$27.449x_{12} - 21.186$	0.534 (18)	0.482 (18)	0.232 (18)	0.185 (18)	0.322 (18)	-2.150 (17)	-2.104 (17)	-1.968 (1)	-2.032 (15)	p2
W_B7	$21.513\ln x_{12} + 5.571$	0.528 (19)	0.476 (19)	0.234 (19)	0.185 (19)	0.324 (19)	-2.138 (19)	-2.092 (18)	-1.956 (2)	-2.020 (16)	p2
W_B8	$-\frac{16.854}{x_{12}}+21.836$	0.523 (20)	0.470 (20)	0.236 (20)	0.185 (20)	0.326 (20)	-2.126 (20)	-2.080 (19)	-1.944 (3)	-2.008 (17)	p2

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Table 20. Best models of the number of PPP contracts in Belgium between 2009 and 2019

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
L_B1	$0.235x_3+0.041x_4-0.133x_8+0.464x_{10}-28.039$	0.828 (1)	0.714 (1)	0.284 (1)	0.277 (6)	0.525 (1)	-1.099 (1)	-0.985 (1)	0.106 (1)	-0.804 (1)	p2
L_B2	$0.154x_3-0.144x_8+0.447x_{10}-5.553x_{11}+0.345x_{13}+12.070$	0.819 (2)	0.639 (2)	0.323 (2)	0.229 (2)	0.590 (2)	-0.889 (2)	-0.752 (2)	1.157 (10)	-0.535 (3)	p1
L_B3	$0.229x_3-0.121x_8+0.462x_{10}+6.713x_{14}-87.893$	0.777 (5)	0.629 (3)	0.331 (3)	0.306 (9)	0.598 (3)	-0.725 (3)	-0.725 (3)	0.366 (4)	-0.544 (2)	p2
L_B4	$0.038x_2+0.257x_3-0.105x_8+0.642x_{10}-44.243$	0.767 (6)	0.612 (4)	0.350 (4)	0.299 (8)	0.612 (4)	-0.680 (4)	-0.680 (4)	0.410 (5)	-0.500 (4)	p2
L_B5	$0.496x_3-0.149x_7-9.736x_{11}\pm65.278x_{12}+0.481x_{13}+89.000$	0.802 (3)	0.603 (5)	0.355 (5)	0.244 (3)	0.618 (5)	-0.660 (5)	-0.660 (5)	1.249 (11)	-0.443 (5)	p1
L_B6	$0.220x_3+0.068x_5-0.276x_6-0.099x_8+0.362x_{10}-9.519$	0.794 (4)	0.588 (6)	0.370 (6)	0.218 (1)	0.630 (6)	-0.622 (6)	-0.622 (6)	1.287 (12)	-0.405 (7)	p1

D – designation of the model. Variable designations as in Table 2. In parentheses the position according to the criterion.
b – significance criterion satisfied.

Table 21. Summary of number of contract models and PPP values per country and number of occurrences of each explanatory variable in the models

Country	Criterion	Number of models	Designation of explanatory variables (as per Table 2)													
			1	2	3	4	5	6	7	8	9	10	11	12	13	14
			number of appearances													
Value of PPP contracts																
France	p1	22	0	0	0	0	2	2	3	0	0	13	8	6	3	4
France	p2	15	0	0	0	0	2	0	0	0	0	8	8	6	0	2
Great Britain	p1	24	2	9	8	1	8	12	12	1	9	0	5	4	6	6
Great Britain	p2	6	0	0	3	0	3	3	2	0	4	0	1	2	0	2
Germany	p1	53	11	13	0	12	11	19	1	0	8	12	17	47	11	13
Germany	p2	14	0	6	0	0	4	3	1	0	0	1	4	14	0	6
Netherlands	p1	15	3	4	0	9	3	1	1	1	3	10	7	5	2	0
Belgium	p1	20	0	4	7	4	0	0	12	11	0	11	5	14	8	1
Belgium	p2	14	0	1	6	2	0	0	8	5	0	5	1	8	7	1
Number of PPP contracts																
France	p1	33	4	21	2	4	11	6	6	10	3	4	1	2	3	3
France	p2	9	0	5	0	0	3	3	2	3	0	0	0	0	0	0
Great Britain	p1	45	4	16	17	3	16	21	18	3	8	1	10	18	5	4
Great Britain	p2	6	0	1	2	0	0	3	0	0	0	0	0	1	3	0
Germany	p1	23	5	4	0	3	1	1	2	13	3	3	17	15	2	0
Germany	p2	6	2	3	0	1	0	0	0	6	0	0	5	5	0	0
Netherlands	p1	48	10	3	1	9	7	6	4	27	6	0	4	36	18	10
Netherlands	p2	5	0	0	1	0	1	2	1	3	0	0	0	4	0	0
Belgium	p1	13	0	3	12	2	1	1	1	12	2	9	4	1	4	2
Belgium	p2	3	0	1	3	1	0	0	0	3	0	3	0	0	0	1

For the number of PPP contracts, 13 models were obtained, including 3 models satisfying the p2 criterion. These models are:
- 4 linear models with five variables,
- 7 models with four variables (3 linear models, 3 logarithmic models and hyperbolic I model),
- linear and log-linear model with variables 3, 8 and 9.

The best model is shown in Table 20. The best model, according to all criteria except the SMAD criterion (according to this criterion it is the sixth best model), is the linear model with variables 3, 4, 8, 10 (L_B1). The second best linear model with variables 3, 8, 10, 11, 13 (L_B2)[19]. In the third and fourth positions, linear models with variables 3, 8, 10, 14 (L_B3) and 22, 3, 8, 10 (L_B4). Models L_B1, L_B3, L_B4 satisfy the p2 criterion. Variable 1 does not appear as an explanatory variable among the models satisfying the p1 criterion. Variables 3 and 8 occur in 12 models (in 11 simultaneously). Models satisfying criterion p2 use variables 2, 3, 4, 8, 10. Variables 3, 8, 10 occur in all models.

Table 21 presents a summary of the number of models of value and number of agreements satisfying the p1 and p2 criteria for particular countries, as well as the number of occurrences in the models of particular explanatory variables. Taking the number of occurrences (or, more precisely, the frequency) of a variable in the models as a criterion, one can assess the importance of a given variable for the formation of the value and the number of PPP agreements. It is not a perfect measure – it may happen that a frequently occurring variable does not appear in the model(s) of the best[20].

Considering all countries and based on the best models, it can be concluded that for:
- the value of PPP contracts:
  - the variables of the greatest importance are[21]:
    - 8, 11, 12 when considering models satisfying condition p1,
    - 12, 7, 11 when considering models satisfying condition p2 only,
  - the variables of the least importance are[22]:
    - by 1, 14 when considering models satisfying condition p1,
    - by 1, 2, 4, 6, 14 when considering models satisfying condition p2 only,
- number of PPP agreements:

[19] This model is positioned at position ten according to the AICc criterion and the BIC criterion at position three.
[20] E.g. variables 7 and 2 for the value of PPP contracts in the GB.
[21] They are most commonly found in models.
[22] Not found in models.

- variables are the most important:
  - by 8, 2, 12, 13 when considering models satisfying condition p1,
  - by 8, 12 when considering models satisfying condition p2 only,
- variables are the least important:
  - by 1, 9, 14 when considering models satisfying condition p1,
  - at 1, 9, 11, 13, 14 when only models satisfying condition p2 are considered.

Stage II

For the sake of homogeneity, 5 principal components are included, which ensures that more than 97% of the variability is accounted for, both for the components based on the covariance matrix and the matrix of linear correlation coefficients. This is due to the fact that for the fifth component based on the matrix of linear correlation coefficients for Belgium, the percentage of explained variability (PEV) is 5% (Table 35).

Regression models with principal components based on the covariance matrix as explanatory variables

The eigenvalues of the vectors and their contribution to explaining the observed variability are presented in Table 22, the eigenvectors are presented in Table 23. Similarly to stage I, the slope of the regression lines and the values of the R^2 coefficient were calculated (Table 24). The ranges of variability of the components are given. The fit in most cases is weak or very weak. Only in 3 cases it exceeds 0.4 (it concerns the fifth components for the GB and the fourth for Germany – the model of the number of contracts). Apart from five cases (out of 50), the significance of p is greater than 0.1. No autocorrelation of residuals is found. There is a change of signs of some slops:

– between individual components between models of number and value of contracts for the same country (except for the GB),
– between individual components between individual countries.

Table 24 gives the value of the increment of the explanatory variable when the value of the principal components increases by a 5% spread (and the range for each principal component). Using these values, it can be concluded that the strongest effect is of the principal components[23]:

– on the value of contracts :
  - France: 2(n), 3(n), 1(n),
  - Great Britain: 5(n), 2(n), 3(p),
  - Germany: the strongest influence (positive) is from component 1,
  - Netherlands: largest effect (positive) is from component 5,
  - Belgium: the highest influence (positive) is from component 5,
– on the number of contracts:
  - France: 2(n), 1(n), 3(n),
  - Great Britain: 5(n), 2(n),
  - Germany: 4(n),
  - Netherlands: 3(p),
  - Belgium: the highest influence (positive) is exerted by component 4.

France. Only one model (WZ_F1, Table 25) satisfying the p1 significance criterion was obtained. This model also satisfies the p2 criterion. It is a hyperbolic I model based on the fourth component. The situation is analogous for the number of contracts. The LZ_F1 model (Table 26) is the only model satisfying the significance criterion p1. It also satisfies criterion p2.

Great Britain. Eight models were obtained, of which four models satisfying the p2 criterion. These are:

– hyperbolic model I with components 1, 2, 4 and 5,
– linear model with components 2, 3 and 5,
– 4 models with two components (two linear models and two hyperbolic I models),
– linear model with component 5,
– hyperbolic I model with component 4.

Table 22. Eigenvalues of principal components and their contribution to explaining observed variability. Components based on the covariance matrix

D	France			Great Britain			Germany			Netherlands			Belgium		
	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]
1	127434	96.24	96.24	231929	80.82	80.82	576436	93.96	93.96	61274.1	96.48	96.48	6015.66	81.69	81.69
2	4142.61	3.13	99.37	40379.3	14.07	94.89	23293	3.80	97.75	1427.97	2.25	98.73	1006.19	13.66	95.36
3	564.129	0.43	99.79	11095.4	3.87	98.76	12321	2.01	99.76	685.85	1.08	99.81	147.885	2.01	97.36
4	213.042	0.16	99.95	3006.32	1.05	99.81	1027.06	0.17	99.93	69.6458	0.11	99.92	130.295	1.77	99.13
5	48.1685	0.04	99.99	439.932	0.15	99.96	367.83	0.06	99.99	38.2323	0.06	99.98	47.1852	0.64	99.77

D – number of principal components z, E – eigenvalue, PEV – percent of explained variation,
CPEV – cumulated percent of explained variation.

[23] Absolute value of increment above EUR 0.1 billion or number of contracts.

Table 23. Eigenvectors. Components based on the covariance matrix

D	Explanatory variables													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	France													
1	0.422769	0.117979	-0.069625	0.706932	-0.279389	-0.009583	0.003863	0.045834	0.471673	-0.000112	0.000047	-0.000008	0.001046	0.002243
2	-0.031032	-0.122455	0.075658	-0.327451	-0.932238	-0.041544	-0.003733	-0.014090	0.008757	0.001971	0.001015	0.000036	-0.001146	-0.001939
3	-0.518723	-0.391914	-0.179588	0.580217	-0.153020	-0.020087	-0.039205	0.029345	-0.426747	0.006852	0.001354	0.000213	-0.004463	0.005229
4	-0.612899	0.498321	-0.382890	-0.035856	-0.055502	-0.013202	-0.054753	-0.219904	0.410501	0.051849	-0.001635	0.001364	0.048221	0.001160
5	-0.366940	0.049739	0.879694	0.152294	0.025819	0.004726	-0.041792	0.005902	0.233417	0.030573	0.003738	0.000318	-0.089002	0.000419
	Great Britain													
1	-0.599655	-0.091778	0.073914	-0.753189	0.210154	0.004635	-0.006426	0.022194	0.109760	0.049504	-0.000244	0.000001	-0.002784	-0.002877
2	0.077858	-0.006214	-0.051976	0.108290	0.874119	0.037437	0.008044	-0.011367	-0.461484	-0.027660	0.000155	0.000032	0.007573	0.001400
3	-0.335799	0.054560	-0.138694	0.007747	-0.417225	-0.023135	0.008035	-0.006527	-0.830486	-0.022816	0.000551	0.000039	0.003063	0.003101
4	-0.686815	0.314907	0.135876	0.588660	0.116584	0.001962	0.016692	0.006160	0.221906	0.029094	-0.000601	0.000009	-0.010654	-0.004641
5	0.009811	0.485327	-0.850550	-0.116477	0.042849	0.024873	0.003371	-0.047513	0.146264	0.030801	0.001861	-0.000006	0.004015	0.016535
	Germany													
1	0.415864	0.189312	-0.061077	0.024264	0.880360	0.023339	0.002442	-0.002010	0.105793	-0.010805	0.000115	0.000009	0.000478	0.000843
2	-0.421927	-0.439644	0.125693	-0.703401	0.302990	0.011000	-0.005466	0.020787	0.158445	0.024892	0.000801	-0.000013	0.000091	0.002951
3	-0.574160	-0.200975	0.038573	0.525088	0.358621	0.028603	-0.011293	-0.014971	-0.472158	-0.000469	-0.000176	-0.000027	-0.003768	-0.004221
4	-0.277553	0.119325	0.456385	0.361015	0.037699	-0.016152	-0.000119	0.026269	0.752401	0.037048	-0.002205	0.000028	-0.015767	0.000883
5	0.395997	-0.294534	0.813442	0.010039	-0.030992	-0.010052	0.007052	0.020054	-0.303782	-0.016023	0.000019	-0.000056	0.012057	0.021191
	Netherlands													
1	-0.234039	-0.296097	0.070471	-0.033400	-0.884010	-0.109290	0.008013	0.000775	-0.240517	0.007338	0.000387	0.000026	0.009151	-0.001005
2	0.250796	-0.470382	-0.026112	-0.769705	0.131569	-0.046456	-0.004395	-0.315101	-0.029865	-0.048813	0.003682	0.000090	0.025476	-0.000310
3	-0.271990	-0.773103	-0.001563	0.374537	0.350524	0.090113	-0.030013	0.168477	-0.165018	0.023380	0.000348	0.000101	0.001426	-0.000453
4	-0.008269	0.232293	-0.067159	-0.205344	0.149205	0.070217	-0.151532	0.313134	-0.855352	-0.128836	-0.005958	-0.000324	0.053159	-0.000040
5	-0.445077	0.125815	0.799106	-0.125743	0.160295	-0.068856	-0.184154	-0.224720	-0.028763	0.086712	0.001176	-0.000405	0.092103	-0.004074
	Belgium													
1	-0.526003	-0.465287	0.057070	-0.506647	-0.152793	0.029061	0.007267	-0.081863	-0.464224	-0.011039	0.002012	0.000019	-0.017273	-0.002899
2	0.014095	-0.301986	-0.003273	-0.123018	0.907831	0.217107	0.002041	0.079423	0.121433	0.027410	-0.001086	0.000034	-0.018919	-0.000735
3	-0.315382	0.526718	0.058589	0.235534	0.263575	0.055343	0.464405	0.071999	-0.510256	-0.069909	0.000624	0.000134	0.074521	0.001096
4	0.170387	0.460968	-0.018540	-0.394471	0.169978	0.023157	-0.403942	-0.605297	-0.176149	-0.082552	-0.000542	0.000063	-0.075491	-0.001118
5	-0.335045	-0.051284	-0.131555	0.531448	0.076859	0.025090	-0.723307	0.087445	-0.216966	0.013233	0.000830	0.000777	0.027621	0.000819

D – number of principal components z. Denotation of explanatory variables according to Table 2.

Table 24. Slope and R^2 coefficients of determination of linear regressions of the value and number of PPP contracts. Principal components based on the covariance matrix

D	France				Great Britain				Germany				Netherlands				Belgium			
	R^2	a	b	ΔW	R^2	a	b	ΔW	R^2	a	b	ΔW	R^2	a	b	ΔW	R^2	a	b	ΔW
contract values																				
1	0.111	-0.00215	I	-0.1195	0.011	0.00034	I	0.0255	0.324	0.00035	?	0.0385	0.237	-0.00116	?	-0.0477	0,037	-0,00112	I	-0.0137
2	0.181	-0.01522	I	-0.1606	0.235	-0.00381	I	-0.1313	0.114	0.00104	I	0.0275	0.001	0.00059	I	0.0035	0,003	-0,00080	I	-0.0035
3	0.270	-0.05041	I	-0.1538	0.211	0.00689	I	0.1113	0.004	-0.00026	?	-0.0054	0.018	-0.00300	I	-0.0117	0,005	-0,00268	I	-0.0056
4	0.018	0.02121	I	0.0456	0.033	0.00525	I	0.0533	0.013	-0.00165	?	-0.0089	0.007	-0.00581	I	-0.0073	0,025	0,00622	I	0.0128
5	0.076	-0.09145	?	-0.0960	0.438	-0.04987	?	-0.1870	0.000	-0.00022	?	-0.0006	0.048	0.02090	I	0.0219	0,199	0,02902	I	0.0348
number of contracts																				
1	0.191	-0.00258	I	-0.1434	0.124	0.00110	I	0.0818	0.000	-0.00003	?	-0.0033	0.087	-0.00154	I	-0.0634	0.017	-0.00166	I	-0.0203
2	0.188	-0.01423	?	-0.1501	0.251	-0.00376	?	-0.1295	0.190	0.00294	?	0.0778	0.076	-0.00943	I	-0.0561	0.006	0.00236	I	0.0101
3	0.164	-0.03597	?	-0.1097	0.110	0.00475	I	0.0767	0.001	0.00034	?	0.0071	0.283	0.02625	I	0.1024	0.004	-0.00486	I	-0.0102
4	0.091	0.04351	I	0.0935	0.002	0.00134	I	0.0136	0.429	-0.02098	?	-0.1133	0.094	0.04757	I	0.0595	0.120	0.02979	I	0.0611
5	0.014	0.03542	?	0.0372	0.460	-0.04875	?	-0.1828	0.003	-0.00283	I	-0.0083	0.005	0.01525	I	0.0160	0.004	0.00855	I	0.0103
range of variation of principal components																				
	min	max			min	max			min	max			min	max			min	max		
1	-549	563			-595	893			-951	1248			-467	356			-115	130		
2	-95	116			-269	420			-162	367			-44	75			-46	40		
3	-32	29			-204	119			-117	299			-31	47			-21	21		
4	-17	26			-111	92			-50	58			-15	10			-23	18		
5	-13	8			-43	32			-21	38			-12	9			-9	15		

D – number of principal component z. R^2 – coefficient of determination. a – slope of the regression line. b – Durbin – Watson test result: I – no autocorrelation of residuals, ? – inconclusive result. $\Delta W = a(\max(z) - \min(z))$ – the increase in the value of the agreements with an increase in the principal component by 5% of the variance. Significance of directional coefficients for principal components: for value of contracts criterion p2 satisfies component 5 for the GB, criterion p1 satisfies component 1 for Germany; for number of contracts: criterion p2 satisfies component 5 for the GB and component 4 for Germany, criterion p1 satisfies component 3 for the Netherlands.

Table 25. Best models of the value of PPP contracts in France between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
WZ_F1	$\frac{7,633}{z_4} + 1,21183$	0.514	0.460	1.194	1.007	1.693	1.171	1.216	1.353	1.289	p2

D – model designation. Designation of variables as in Table 23. b – satisfied significance criterion.

Table 26. Best models of the number of PPP contracts in France between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
LZ_F2	$\frac{6.317}{z_4} + 2.59424$	0.419	0.355	1.154	1.059	1.695	1.173	1.219	1.355	1.291	p2

D – model designation. Designation of variables as in Table 23. b – satisfied significance criterion.

Table 27. Best models of the value of PPP contracts in the Great Britain between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
$WZ_{GB}1$	$-0.00381z_2 + 0.00689z_3 - 0.04987z_5 + 1.43909$	0.884 (1)	0.834 (1)	0.471 (1)	0.214 (1)	0.643 (1)	-0.699 (2)	-0.608 (1)	-0.002 (1)	-0.463 (1)	p2
$WZ_{GB}2$	$\frac{304.786}{z_1} - \frac{87.919}{z_2} + \frac{11.092}{z_4} + \frac{2.607}{z_5} + 1.84800$	0.799 (2)	0.664 (2)	0.565 (2)	0.389 (2)	0.916 (2)	-0.783 (1)	0.127 (2)	1.217 (8)	0.307 (2)	p1
$WZ_{GB}3$	$-0.00381z_2 - 0.04987z_5 + 1.43909$	0,673 (3)	0,591 (3)	0,623 (3)	0,624 (6)	1,010 (3)	0,179 (3)	0,247 (3)	0.559 (3)	0,356 (3)	p2

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

Table 28. Best models of the number of PPP contracts in the Great Britain between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
$LZ_{GB}1$	$0.00110z_1 - 0.00376z_2 + 0.00475z_3 - 0.04875z_5 + 1.45455$	0.945 (1)	0.908 (1)	0.310 (1)	0.141 (1)	0.458 (1)	-1.501 (1)	-1.260 (1)	-0.169 (1)	-1.079 (1)	p2
$LZ_{GB}2$	$\frac{172.659}{z_1} - \frac{81.330}{z_2} + \frac{12.478}{z_4} + \frac{2.849}{z_5} + 2.08346$	0.892 (2)	0.820 (2)	0.401 (2)	0.262 (2)	0.639 (2)	-1.374 (2)	-0.592 (2)	0.499 (5)	-0.411 (2)	p2
$LZ_{GB}3$	$0.00110z_1 - 0.00376z_2 - 0.04875z_5 + 1.45455$	0.834 (3)	0.763 (3)	0.417 (3)	0.431 (4)	0.733 (3)	-0.436 (3)	-0.345 (3)	0.261 (2)	-0.200 (3)	p1

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance.

The best models are presented in Table 27. The best model (according to all criteria except the HQC criterion, which places the model in second position) is a linear model with components 2, 3, 5 ($WZ_{GB}1$). It is also the best model satisfying criterion p2. Components 2 and 5 are present in five of the eight models, including three of the four that satisfy the p2 criterion.

For the number of contracts, 9 models were obtained, including 6 models satisfying the p2 criterion. These are:
- linear model and a hyperbolic I model with four variables,
- 2 linear models with three variables,

- 3 models with two variables (a linear model and two hyperbolic I models),
- linear model with component 5,
- hyperbolic I model with component 4.

The best models are shown in Table 28. The best model (according to all criteria) is a linear model with components 1, 2, 3 and 5 ($LZ_{WB}1$). It is also the best model satisfying criterion p2. Component 2 is found in seven models, including five of the six that satisfy criterion p2. Component 5 occurs in six of the nine models, including four of the six that satisfy the p2 criterion.

Table 29. Best models of the value of PPP contracts in Germany between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
WZ_D1	$0.00035z_1+0.51455$	0.324	0.248	0.342	0.139	0.405	-1.690	-1.645	-1.508	-1.572	p1

D – model designation. Designation of variables as in Table 23. b – satisfied significance criterion.

Table 30. Best models of the number of PPP contracts in Germany between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
LZ_D1	$0.00294z_2-0.02098z_4+1.36364$	0.619 (1)	0.524 (1)	0.474 (2)	0.393 (3)	0.709 (1)	-0.530 (1)	-0.462 (1)	-0.150 (1)	-0.354 (1)	p1
LZ_D2	$-0.02098z_4+1.36364$	0.429 (2)	0.365 (2)	0.470 (1)	0.357 (1)	0.818 (2)	-0.284 (3)	-0.238 (2)	-0.102 (2)	-0.166 (2)	p2
LZ_D3	$\frac{370,412}{z_1}-\frac{4.645}{z_5}+1.241644$	0,369 (3)	0,211 (3)	0,682 (3)	0,392 (2)	0,912 (3)	-0,025 (3)	0,043 (3)	0.355 (3)	0,152 (3)	p1

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

Table 31. Best models of the number of PPP contracts in the Netherlands between 2009 and 2019. Principal components based on the covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
LZ_N1	$0.02625z_3+1.54545$	0.283	0.203	0.897	0.561	1.155	0.405	0.451	0.451	0.523	p1

D - model designation. Designation of variables as in Table 23. b - satisfied significance criterion.

Table 32. Best models of the value of PPP contract values in Belgium between 2009 and 2019. Principal components based on covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
WZ_B1	$-\frac{0.266}{z_3}+0.17710$	0.709 (1)	0.677 (1)	0.142 (1)	0.190 (1)	0.254 (1)	-2.622 (1)	-2.576 (1)	-2.440 (1)	-2.504 (1)	p2
WZ_B2	$\frac{0.816}{z_1}+0.18907$	0.667 (2)	0.630 (2)	0.156 (2)	0.200 (2)	0.272 (2)	-2.485 (2)	-2.439 (2)	-2.303 (2)	-2.367 (2)	p2

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

Table 33. Best models of the number of PPP contract values in Belgium between 2009 and 2019. Principal components based on covariance matrix

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
LZ_B1	$-\frac{0.299}{z_4}+1.25945$	0.333	0.259	0.583	0.483	0.845	-0.217	-0.173	-0.037	-0.101	p2

D – model designation. Designation of variables as in Table 23. b – satisfied significance criterion.

Germany. Only one model was obtained for the value of contracts (WZ_D1, Table 29). It is a linear model based on the first component. It satisfies only the p1 criterion. The fit is poor ($R^2 = 0.324$).

For the number of contracts (Table 30), only three models were obtained, including one that satisfies the p2 criterion (LZ_D2). This is a linear model with a fourth component as an explanatory variable. Not all criteria indicate the same model as the best (both second and third position). However, considering all criteria, the linear model with variables 2 and 4 (LZ_D1) should be considered as the best. The second position is occupied by the LZ_D2 model and the third position by the hyperbolic I model with variables 1 and 5 (LZ_D3).

Netherlands. In the case of the Netherlands, there are no models for the value of PPP contracts that satisfy criterion p1 (so also criterion p2). For the number of contracts, there is only one model satisfying p1 (Table 31). It is based on the third component. The fit of this model is very poor ($R^2 = 0.283$).

Belgium. There are two models for contract values. These are hyperbolic I models with one variable based on the third (WZ_B1) and first (WZ_B2) principal components (Table 32). These models satisfy the p2 criterion.

For the number of contracts, only one model was obtained (LZ_B1, Table 33). The model satisfies only the p1 criterion. It is a hyperbolic I model based on the fourth component.

Regression models with principal components based on a matrix of linear correlation coefficients as explanatory variables

Eigenvalues of the vectors and their contribution to explaining the observed variability are in Table 34, eigenvectors are in Table 35. Slops of regression lines, R^2 measures and ranges of variation of the components are in Table 36. The fit in most cases is weak or very weak. Only in 2 cases it exceeds 0.4 (number of contracts – second component for GB and fourth for Belgium). Apart from four cases, the significance of p is greater than 0.1. No autocorrelation of the residuals is found. There is a change of signs of some coefficients:

- between individual components between models of number and value of contracts for the same country (except for the GB),
- between individual components between individual countries.

Table 36 shows the value of the increment of the explanatory variable when the value of the principal components increases by 5% of the variance (and the variance for each principal component). Using these values, it can be concluded that the strongest effect is of the principal components[24]:

- on the value of contracts :
  - France: 3(p), 1(n),
  - Great Britain: 2(n),
  - Germany: the greatest influence (positive) is exerted by component,
  - Netherlands: the greatest influence (negative) is exerted by component 1,
  - Belgium: the greatest influence (positive) is exerted by component 4,
- on the number of contracts:
  - France: 1(n), 4(p), 3(p),
  - Great Britain: 3(p),
  - Germany: 5(p),
  - Netherlands: 4(p),
  - Belgium: highest impact (positive) from component 4.

Table 34. Eigenvalues of principal components and their contribution to explaining observed variability. Components based on a matrix of linear correlation coefficients

D	France			Great Britain			Germany			Netherlands			Belgium		
	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]	E	PEV [%]	CPEV [%]
1	8.6038	61.46	61.46	8.7750	62.68	62.68	8.3390	59.56	59.56	8.6284	61.63	61.63	7.9515	56.80	56.80
2	2.4704	17.65	79.10	2.9852	21.32	84.00	3.2010	22.86	82.43	2.9246	20.89	82.52	2.3179	16.56	73.35
3	1.6513	11.79	90.90	0.9440	6.74	90.74	1.0044	7.17	89.60	1.0893	7.78	90.30	1.5232	10.88	84.23
4	0.5705	4.07	94.97	0.5776	4.13	94.87	0.7858	5.61	95.21	0.7585	5.42	95.72	1.1438	8.17	92.40
5	0.3482	2.49	97.46	0.3033	2.17	97.04	0.3477	2.48	97.70	0.3297	2.35	98.07	0.7003	5.00	97.40

D – principal components number, E – eigenvalue, PEV – percent of explained variation,
CPEV – cumulated percent of explained variation.

[24] Absolute value of increment above EUR 0.1 billion or number of contracts.

Table 35. Eigenvectors. Components based on a matrix of linear correlation coefficients

D	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	France													
1	0.33901	0.32069	-0.31401	0.33681	-0.29587	-0.27115	0.23915	0.33113	0.33929	-0.02306	0.04352	-0.04729	0.12167	0.33901
2	0.04157	-0.04579	0.12712	-0.00691	0.07747	0.12411	0.35988	0.09606	-0.01949	-0.59749	-0.06626	-0.59061	-0.32891	0.04157
3	0.00310	0.14857	-0.15625	0.01584	0.27616	0.33075	0.21534	-0.04788	0.00871	0.05429	-0.72282	-0.03515	0.43781	0.00310
4	0.06454	0.20108	-0.15384	0.18190	0.37070	0.44930	-0.21887	0.15099	0.06475	-0.00023	0.00648	0.28231	-0.56261	0.06454
5	-0.03584	-0.08388	-0.29184	0.02717	0.24354	0.23330	-0.28300	0.08591	-0.07997	-0.39912	0.46494	-0.09375	0.54721	-0.03584
	Great Britain													
1	-0.31115	-0.29165	0.31387	-0.32018	0.10366	0.03184	-0.29700	0.33039	0.21457	0.33133	-0.31049	-0.05749	-0.21147	-0.32822
2	-0.11576	-0.17077	-0.04545	-0.11497	0.52114	0.52750	0.08153	0.01686	-0.33252	0.01796	0.03412	0.43462	0.29051	-0.00570
3	-0.27651	-0.09482	-0.19494	-0.18278	-0.26237	-0.36867	-0.04081	0.09580	-0.36923	0.12182	0.27157	0.51480	-0.35275	0.11481
4	0.15142	-0.37954	-0.17975	-0.05026	-0.18632	-0.12755	-0.57604	-0.00807	0.19464	0.02247	0.17490	0.10140	0.56855	0.11317
5	0.15104	0.08633	0.26325	0.27088	-0.09027	-0.02057	-0.00294	-0.21358	0.40690	-0.04909	-0.32441	0.70254	-0.07291	-0.02797
	Germany													
1	0.34477	0.33099	-0.31363	0.06490	0.32470	0.31108	0.30952	-0.14933	0.27667	-0.32634	0.14643	0.26219	0.17141	0.21472
2	0.01507	0.14315	-0.14306	0.53697	-0.04640	-0.02831	0.00967	-0.46461	-0.25608	-0.17043	-0.41023	0.03949	-0.18763	-0.38974
3	-0.04482	-0.05434	-0.10166	0.04566	0.03142	0.09953	-0.19966	-0.32583	-0.33046	-0.08848	0.45490	-0.42468	0.56052	-0.08486
4	0.01043	-0.05819	-0.01892	0.03741	0.36388	0.46237	-0.37160	0.02389	-0.00904	-0.02261	0.25034	-0.23005	-0.62571	-0.04685
5	-0.10767	-0.14002	-0.29789	-0.25479	0.01653	0.04357	-0.25187	-0.02048	-0.08201	0.10607	0.21844	0.67660	0.06681	-0.47029
	Netherlands													
1	-0.32703	-0.32746	0.32672	-0.11442	-0.33558	-0.33693	0.20240	-0.02718	-0.33315	0.20140	0.19950	0.12940	0.27437	-0.33879
2	-0.15774	0.04242	0.07950	0.54172	-0.05869	-0.00277	0.03814	0.56118	-0.05101	0.41380	-0.39236	-0.05824	-0.14664	-0.03227
3	0.00741	-0.01754	-0.10792	0.00946	0.00740	0.00368	0.28396	-0.11050	0.04197	0.10720	0.02306	0.80307	-0.48570	0.01947
4	-0.05562	-0.13320	0.08775	0.00624	0.00275	0.01709	-0.83192	0.09292	-0.11329	-0.15085	-0.16737	0.42984	0.13441	-0.06304
5	-0.01895	0.12335	-0.05783	-0.25920	-0.17307	-0.17213	0.27175	0.10163	-0.18456	-0.50939	-0.68579	0.04734	0.05068	-0.03840
	Belgium													
1	0.34454	0.32131	-0.31475	0.34805	0.13807	-0.09991	-0.00357	0.25871	0.34610	0.19725	-0.33726	-0.07474	0.25075	0.34780
2	0.01966	0.21555	0.01657	0.09156	-0.56374	-0.60697	0.03332	-0.19079	-0.05742	-0.38002	0.10255	-0.01926	0.22862	0.09954
3	-0.12909	-0.06927	0.21776	0.00424	-0.00402	0.05255	0.77389	0.36766	-0.10429	0.05400	0.08711	-0.07474	0.40863	-0.01735
4	0.04287	0.08804	0.14650	0.13000	0.14502	0.08522	-0.07753	0.04734	0.00826	-0.15278	0.15440	0.90612	0.19693	0.09172
5	0.10540	0.26656	0.06726	0.00926	0.35499	0.25292	0.28248	-0.37886	-0.02259	-0.63568	-0.25611	-0.14047	-0.06948	0.07181

D – number of principal components s. Designation of explanatory variables according to Table 2.

Table 36. Slope and R^2 coefficients of determination of a linear regression of the value and number of PPP contracts. Principal components based on the matrix of linear correlation coefficients

D	France				Great Britain				Germany				Netherlands				Belgium			
	R^2	a	b	ΔL	R^2	a	b	ΔL	R^2	a	b	ΔL	R^2	a	b	ΔL	R^2	a	b	ΔL
contract values																				
1	0.099	-0.23510	l	-0.1079	0.024	0.03475	l	0.0153	0.304	0.08502	?	0.0370	0.187	-0.07853	l	-0.0406	0.021	0.02169	?	0.0087
2	0.047	0.30289	l	0.0706	0.419	-0.64965	l	-0.1848	0.104	-0.08020	?	-0.0258	0.016	0.03939	l	0.0101	0.006	0.02206	l	0.0050
3	0.235	0.82942	l	0.1646	0.038	0.06860	?	0.0123	0.005	-0.03008	?	-0.0051	0.179	0.21630	l	0.0361	0.081	-0.09828	l	-0.0208
4	0.001	0.10821	l	0.0141	0.183	-0.17902	l	-0.0255	0.001	0.01149	?	0.0021	0.004	0.03880	l	0.0060	0.566	0.31609	l	0.0575
5	0.004	0.22107	l	0.0224	0.070	0.07028	l	0.0071	0.329	0.43327	l	0.0487	0.002	0.04145	l	0.0030	0.018	-0.06902	l	-0.0088
number of contracts																				
1	0.215	-0.31773	l	-0.1458	0.211	0.22296	l	0.0981	0.000	-0.00507	?	-0.0022	0.098	-0.1223	?	-0.0633	0.003	0.01904	l	0.0076
2	0.000	-0.02540	?	-0.0059	0.119	-0.28705	l	-0.0817	0.126	-0.19425	?	-0.0625	0.269	0.3485	l	0.0897	0.000	-0.00232	l	-0.0005
3	0.121	0.54368	?	0.1079	0.195	0.65369	l	0.1177	0.166	0.39808	n	0.0673	0.084	0.3201	l	0.0535	0.056	-0.17868	l	-0.0379
4	0.152	1.03889	?	0.1351	0.006	-0.14472	l	-0.0206	0.003	-0.06245	?	-0.0112	0.289	0.7100	l	0.1090	0.113	0.31015	?	0.0564
5	0.053	-0.78294	?	-0.0795	0.070	0.69231	l	0.0703	0.384	1.02901	l	0.1158	0.001	-0.0720	l	-0.0053	0.032	0.20538	l	0.0262
range of variation of principal components																				
	min	max			min	max			min	max			min	max			min	max		
1	-4.37	4.81			-3.33	5.47			-4.27	4.44			-5.11	5.24			-4.5	3.51		
2	-3.05	1.61			-2.59	3.1			-4.35	2.08			-3.05	2.1			-2.31	2.18		
3	-2.17	1.8			-2.41	1.19			-1.35	2.03			-2.23	1.11			-1.74	2.5		
4	-1.39	1.21			-1.53	1.32			-1.42	2.16			-1.15	1.92			-1.28	2.36		
5	-1.06	0.97			-1.04	0.99			-1.27	0.98			-0.7	0.77			-1.29	1.26		

D – number of the principal component variable s. R^2 – coefficient of determination. a – slope of the regression line. b – Durbin – Watson test result: l – no autocorrelation of residuals, ? – inconclusive result. ΔL = a(max(z) – min(z)) – the increase in the value of the agreements with an increase in the principal component by 5% of the variance. Significance of the directional coefficients for the principal components: for the value of contracts criterion p1 is satisfied by components 1 and 5 for Germany; for the number of contracts: criterion p2 is satisfied by component 5 for Germany and the Netherlands and component 4 for Belgium.

Table 37. Best models of the value of PPP contracts in France between 2009 and 2019. Principal components based on a matrix of linear correlation coefficients

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
WS_F1	$\frac{0.099}{s_3}-\frac{0.483}{s_4}+\frac{0.283}{s_5}-0.35151$	0.719 (1)	0.599 (1)	0.991 (1)	0.640 (1)	1.459 (1)	0.303 (1)	1.030 (1)	0.621 (1)	1.175 (1)	p1
WS_F2	$-\frac{0.395}{s_4}+0.32640$	0.465 (2)	0.405 (2)	1.327 (2)	0.952 (2)	1.777 (2)	0.949 (2)	1.313 (2)	1.586 (2)	1.385 (2)	p2

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

Table 38. Best models of the value of PPP contracts in the Great Britain between 2009 and 2019. Principal components based on a matrix of linear correlation coefficients

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
$WS_{WB}1$	$-\frac{0.650}{s_2}+1.29793$	0.432	0.369	0.890	0.740	1.255	0.572	0.618	0.754	0.690	p2

D – model designation. Designation of variables as in Table 23. b – satisfied significance criterion.

Table 39. Best models of the value of PPP contracts in Germany between 2009 and 2019. Principal components based on a matrix of linear correlation coefficients

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
WS_D1	$0.08502s_1-0.43327s_5+0.51455$	0.633 (1)	0.541 (1)	0.232 (1)	0.144 (2)	0.316 (1)	-2.142 (1)	-2.074 (1)	-1.762 (1)	-1.965 (1)	p2
WS_D2	$0.43327s_5+0.51455$	0.329 (2)	0.254 (2)	0.289 (2)	0.234 (3)	0.403 (2)	-1.698 (2)	-1.653 (2)	-1.516 (2)	-1.580 (2)	p1
WS_D3	$0.08502s_1+0.51455$	0.304 (3)	0.226 (3)	0.357 (3)	0.109 (1)	0.411 (3)	-1.662 (3)	-1.616 (3)	-1.480 (3)	-1.544 (3)	p1

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

France. Two models of contract values were obtained, one of which satisfies criterion p2 (Table 37. This is a hyperbolic I model based on component 4 (WS_F2). The best model (according to all criteria) is the hyperbolic I model based on components 3, 4 and 5 (WS_F1).

For the number of contracts, there are no models satisfying criterion p1 (so also p2).

Great Britain. Only one model was obtained. The model satisfies criterion p2. It is a hyperbolic I model ($WS_{GB}1$, Table 38) based on the second component.

For the number of contracts, no models satisfying the p1 criterion.

Germany. For the value of PPP contracts, three models were obtained, including, one satisfying the p2 criterion. These are linear models. The criteria unanimously position the models (Table 39). Only the SMAD criterion sets a different order. The best model is the one with the first and the fifth principal component as explanatory variables (WS_D1). This model simultaneously satisfies the p2 criterion. The second-best model is the model with the fifth principal component (WS_D2). The third position is occupied by the model with the first principal component (WS_D3).

For the number of contracts, three models were also obtained, of which two models satisfy the p2 criterion (Table 40):

- the hyperbolic model I (LSD1), based on components 2 and 3, is the best model according to all criteria,
- linear model I (LS_D2), based on component 5, is second best according to all criteria except SMAD (third best).

The third model is the hyperbolic I model based on the second principal component (LSD3).

Netherlands. For the Netherlands, there are no models for the value of PPP contracts that satisfy the p1 criterion. For the number of contracts, two models satisfying only the p1 criterion were obtained (Table 41). These are:

- the model with the second and fourth components (LS_N1) – the best on all criteria,
- the model with the fourth component (LS_N2).

Belgium. Two models were derived for the value of PPP contracts. They are hyperbolic I models: WS_B1 based on the first principal component and WS_B2 based on the fourth principal component. Both models satisfy the p2 criterion (Table 42).

For the number of contracts, there are no models satisfying the p1 criterion.

Table 40. Best models of the number of PPP contracts in Germany between 2009 and 2019. Principal components based on a matrix of linear correlation coefficients

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
LS_D1	$\frac{0.294}{s_2}+\frac{0.420}{s_3}+1.46313$	0.635 (1)	0.544 (1)	0.458 (1)	0.393 (1)	0.694 (1)	-1.050 (1)	-0.505 (1)	-0.573 (1)	-0.396 (1)	p2
LS_D2	$0.05518s_5+1.33077$	0.384 (2)	0.316 (2)	0.542 (2)	0.571 (3)	0.850 (2)	-0.209 (2)	-0.163 (2)	-0.027 (2)	-0.091 (2)	p2
LS_D3	$\frac{0.370}{s_3}+1.42452$	0.338 (3)	0.265 (3)	0.657 (3)	0.472 (2)	0.880 (3)	-0.137 (3)	-0.092 (3)	0.045 (3)	-0.019 (3)	p1

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance.

Table 41. Best models of the number of PPP contracts in the Netherlands between 2009 and 2019. Principal components based on a matrix of linear correlation coefficients

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
LS_N1	$0.34853s_2+0.70997s_4+1.63636$	0.558 (1)	0.447 (1)	0.621 (1)	0.468 (1)	0.897 (1)	-0.060 (1)	0.009 (1)	0.321 (1)	0.117 (1)	p1
LS_N2	$0.70997s_4+1.63636$	0.289 (2)	0.210 (2)	0.838 (2)	0.511 (2)	1.072 (2)	0.256 (2)	0.302 (2)	0.438 (2)	0.374 (2)	p1

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

Table 42. Best models of the value of PPP contracts in Belgium between 2009 and 2019. Principal components based on a matrix of linear correlation coefficients

D	Equation	R^2	R^2_{adj}	MAD [bln]	SMAD [bln]	SER	HQC	AIC	AICc	BIC	b
WS_B1	$\frac{0.310}{s_1}+1.18182$	0.655 (1)	0.616 (1)	0.162 (1)	0.201 (2)	0.277 (1)	-2.450 (1)	-2.404 (1)	-2.268 (1)	-2.332 (1)	p2
WS_B2	$0.316088s_4+0.299091$	0.566 (2)	0.518 (2)	0.228 (2)	0.172 (1)	0.311 (2)	-2.221 (2)	-2.175 (2)	-2.039 (2)	-2.103 (2)	p2

D – model designation. Designation of variables as in Table 23. In brackets the position according to the given criterion.
b – satisfied significance criterion.

Best models regardless of the type of explanatory variables

Comparing the best models for each country and type of explanatory variable, it should be noted that:

– if we consider models satisfying significance condition p1, then the best models (taking into account all criteria[25]) are the models built based on variables specified in Table 2,
– if we consider models satisfying criterion p2, then:
  ◦ for the number of PPP contracts for the GB, the best model is the LZGB1 model built on the principal components determined from the covariance matrix,
  ◦ in the remaining cases, the best models are those built on the basis of variables specified in Table 2[26].

[25] Incidentally, it can happen that for one of the criteria the classification is reversed, e.g. the SMAD criterion for $W_{GB}1$ and $WZ_{GB}1$ and $W_{GB}2$ and $WZ_{GB}1$.

[26] The situation changes significantly if we strengthen the non-collinearity conditions using Bartlett's sphericity test and assuming p-value is greater than 0.01 for criterion p1 and greater than 0.05 for criterion p2. Then some of the models using the variables specified in Table 2 will not satisfy the modified p1 and p2 conditions. The best models satisfying criterion p1 will be the $WZ_{GB}1$ model for the value of contracts and the $LZ_{GB}1$ and LS_D1 models for the number of contracts. The best models satisfying criterion p2 will be the WS_F1, $WZ_{GB}1$, WZ_B1 models for the value of contracts and the $LZ_{GB}1$ and LS_D1 models for the number of contracts.

Note that it was not always possible to find a model satisfying the p2 criterion or even the weaker p1 condition. Such a situation occurred for:

- condition p2:
  - contract values: for the Great Britain for the variable 's', Germany for the variable 'z', the Netherlands for all variable types,
  - number of contracts: for France for the variable 's', Great Britain for the variable 's', the Netherlands for the variables 's' and 'z', Belgium for the variable 'S',
- condition p1:
  - on contract values: Germany for variables 'z', Netherlands for variables 's' and 'z',
  - on the number of contracts: France for the variable 's', Great Britain for the variable 's', Belgium for the variable 's'

CONCLUSIONS

The standard approach to assessing the power of the influence of a given factor on the studied variable is to determine the slope of the regression line. For the countries considered, and the explanatory variables considered (critical success factors), the cases of significance of the explanatory variables in the regression equations are greater than 0.1. In many cases, there is a change in the sign of the slopes of the regression line. The change occurs both between countries (for the same explanatory variable) and between models of number of contracts and value of contracts (for the same country, excluding the GB). For this reason, as well as the very large difference in the range of values of the different explanatory variables, the assessment of their impact on the value and number of contracts should be approached with caution. The authors suggest that the impact assessment should be based on the coefficients of the explanatory variables in the best-fitting regression models that satisfy certain criteria. In most cases, the best (or one of the best) models (built on the variables defined in Table 2) turned out to be linear models. In addition, hyperbolic I and logarithmic models (for contract values in Germany and the Netherlands at criterion p1). Among the models satisfying criterion p2, linear and hyperbolic I models. Models satisfying the more stringent p2 criterion were not obtained in every case[27]. The best models built based on principal components are linear and hyperbolic I models. In this case also, models satisfying p1 and p2 criteria were not obtained in many cases. The power, hyperbolic II and hyperbolic III models proved to be useless regardless of the type of explanatory variables. The number of PPP contracts is easier to model (using the proposed regression models) than the value of contracts. Principal components based on the covariance matrix generally give better models than those built based on the correlation matrix[28].

If the number of occurrences of a given explanatory variable in the best regression models is taken as a criterion, then:

- variables having a significant effect are (at least two occurrences):
  - when criterion p1 is satisfied:
    - on the value of contracts – 8, 11, 12, 2, 9,
    - on the number of contracts – 2, 8, 12, 13, 3,
  - when criterion p2 is satisfied:
    - on the value of contracts – 12, 7, 11,
    - on the number of contracts – 12, 2, 3, 6, 8,
- variables with insignificant impact (no occurrences):
  - when criterion p1 is satisfied:
    - on the value of contracts – 1, 14,
    - on number of contracts – 1, 9, 14,
  - when criterion p2 is satisfied:
    - on the value of contracts – 1, 2, 4, 6, 14,
    - on the number of contracts – 1, 9, 11, 13, 14.

Important problems with regression models constructed in this way are the inability to consider all variables simultaneously (too few data) and the occurrence of multi-collinearity. Let us note that if we omit the requirement of lack of multi-collinearity, there are linear models (and not only) with 8 explanatory variables satisfying the other conditions of the p2 criterion and R^2 greater than 0.995 (for France and the number of agreements such model is the one with 7 variables and $R^2 = 0.867$). The problem does not occur if we use principal components as explanatory variables in regression models. Due to the values taken by the principal components (positive and negative) some of the models considered could not be used. The best models are linear and hyperbolic I models. There is also no clear answer as to which components are the most significant. What is notable, however, is the presence of a significant proportion of the best component models:

- fourth and third, when the components are based on the covariance matrix,
- third, when the components are based on the correlation matrix.

There is no clear answer to the question of which of the methods of constructing regression models gives better models. The result depends on the adopted

[27] For the value of PPP contracts in the Netherlands.

[28] For the number of contracts, only two models (using principal components based on the correlation matrix) satisfying criterion p1 and one satisfying criterion p2 were obtained.

criteria (the significance of explanatory variables and the lack of multi-collinearity) and the development of the value and number of PPP agreements in a given country. It can be concluded that these methods are complementary.

The paper uses 9 measures (criteria) of model fit. The criteria R^2_{adj}, SER, HQC, AIC_C and BIC show high agreement in the evaluation (positioning) of models. The AICc, SMAD and MAD criteria (to a lesser extent) do not always agree with the other criteria. Note that the best models are linear, hyperbolic I and logarithmic models. For these models, there is SST = SSE+SSR. It seems, therefore, that the best measure of model fit is the criterion R^2_{adj}.

MODELE EKONOMETRYCZNE – METODA BADANIA CZYNNIKÓW REALIZACJI PROJEKTÓW PARTNERSTWA PUBLICZNO-PRYWATNEGO W WYBRANYCH KRAJACH EUROPEJSKICH

Współczesna nauka opiera się na badaniu zjawisk ekonomicznych i stara się je kwantyfikować w sposób wymierny. Do tego celu wykorzystuje się modele ekonometryczne. Przedmiotem badań było opracowanie modeli ekonometrycznych, które pokazują siłę wpływu różnych czynników na realizację projektów partnerstwa publiczno-prywatnego (PPP) w obszarze infrastruktury transportowej w Francji, Wielkiej Brytanii, Niemczech, Holandii i Belgii. Modele te wyrażają zależność wartości i liczby kontraktów PPP od wartości mierzalnych czynników sukcesu PPP. Uwzględniano projekty o wartości co najmniej 40 mln euro. Zastosowano model liniowy oraz siedem modeli przekształcalnych do liniowego. Jako zmienne objaśniające uwzględniono cztery grupy czynników. Uzyskano czternaście wskaźników. Wykorzystano również składowe główne wyznaczane w oparciu o macierze kowariancji i korelacji. Najlepszymi modelami dla liczby umów PPP są modele liniowe i hiperboliczne I. Dla wartości umów – modele liniowe i hiperboliczne I i logarytmiczne. Wskazano modele najlepsze z uwzględnieniem typu zmiennych objaśniających i bez względu na typ zmiennych objaśniających. Do oceny jakości modeli wykorzystano dziewięć kryteriów. Na podstawie modeli najlepszych wskazano czynniki mające istotny wpływ na wartość i liczbę modeli PPP. Wskazano również czynniki nie mające istotnego wpływu.

Słowa kluczowe: model ekonometryczny, regresja liniowa, regresja nieliniowa, partnerstwo publiczno-prywatne PPP

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